THE THEORY OF DIRECT-CURRENT DYNAMOS AND MOTORS

BY

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To G.

PREFACE

UNTIL the present time there has been a remarkable scarcity of English text-books which present, in a single volume, a treatment of the theories and principles underlying the design and use of direct-current generators and motors, in such a form as to meet the needs of University students. Books which have been written for the use of designers are usually too specialised, and the student is apt to be bewildered by a mass of detail, whilst the text-books of general electrical engineering usually treat the matter inadequately. I hope that this book may be found to bridge over the gap which other teachers of electrical theory, besides myself, must have found existing between the two classes of books, and that it will consequently prove useful to students reading for an Honours degree at English universities.

No attempt has been made to explain in detail the construction of dynamos and motors, as that is not part of the function I have set out to perform; moreover, it is a side of the subject which can only be learnt properly by experience in the design, construction, and use of the machines.

The reader is assumed to be familiar with the elementary principles of electricity, magnetism, and electro-magnetism. In particular he is supposed to be conversant with Ohm's Law and the properties of electric circuits, the laws of magnetism and magnetic induction, the magnetization of iron by electric currents including hysteresis, and the relations between the practical system of units, and the absolute (c.g.s.) system.

With this as a foundation, I have tried to develop, step by step, the theory underlying the design, operation, and testing of direct current machinery, beginning with the general theory of induced electromotive forces, upon which rest the main principles of the dynamo machine. I then proceed to apply this theory to the dynamo machine in its capacity as a generator, i.e. a machine for converting mechanical energy into electrical energy, and, afterwards,

pass on to consider it in its reverse aspect, namely, as a motor converting electrical energy into mechanical energy. Having shown how the two functions are performed, and after dealing with the theory of the machine in either capacity, the efficiency of the transformation of energy and the sources of loss of energy are considered. We then pass on to the application of direct current motors to traction, and conclude with a chapter on boosters and multiple wire systems.

In order to illustrate the applications of the various formulae, numerous examples have been worked out in the text, and others have been added as exercises for the student. These examples, for the most part, have been taken from the examination papers of universities and colleges.

I wish to acknowledge my indebtedness to the authors of the many books to which I have referred; and, where direct acknowledgment has not been made in the text, I hope this expression of thanks will suffice.

I also wish to express my gratitude for many valuable suggestions received from Mr. C. G. Lamb and Mr. F. J. Dykes, during the evolution of the book.

My thanks are due to Messrs. The Lancashire Dynamo and Motor Co., Ltd., for characteristic curves and other particulars of dynamos and motors and for the particulars and diagram in § 166, to Messrs. The Electrical Apparatus Co., Ltd., for particulars of motor starters and regulators, and to Messrs. Joseph Sankey & Co. for magnetic curves of iron.

I am also grateful to Mr. B. Low for his assistance in the preparation of many of the diagrams.

JOHN CASE.

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CONTENTS

LIST OF SYMBOLS

The symbols used conform, for the most part, with the list issued by the International Electro-technical Commission in 1914.

$\mathcal{B}$	Flux density, or Magnetic Induction : lines/cm².
E	the value of a steady electro-motive force (E.M.F.)
E_m	the maximum value of a variable E.M.F.
e	the instantaneous value of a variable E.M.F.
F	mechanical force.
$\mathcal{F}$	Magneto-motive force.
f	the number of cycles per second.
$\mathcal{H}$	Magnetic field intensity, or magnetizing force.
h	hysteresis loss in iron in ergs per c.c. per cycle.
I	the value of a steady current.
I_m	the maximum value of a variable current.
i	the instantaneous value of a variable current.
L	coefficient of self-induction.
l	length.
m	mass ; also the number of independent windings on an armature.
N	the number of turns of wire on a coil.
n	revolutions per second.
P	electrical power : watts.
$\mathcal{P}$	permeance = the reciprocal of reluctance.
p	the number of pairs of poles in dynamos and motors.
R, r	resistance.
$\mathcal{R}$	reluctance.
S	the cross section of a magnetic circuit.
s	the cross section of a conductor.
T	the periodic time of an oscillation.
t	time ; also temperature, °C.
U	energy.
V, v	difference of potential (P.D.)
W	work done.
X	reactance.

α	the temperature coefficient of resistance.
γ	conductivity $=\frac{1}{R}$
ϵ	the base of the Naperian logarithms.
η	efficiency; also the Steinmetz hysteretic coefficient.
κ	susceptibility.
μ	permeability.
ν	reluctivity (= the reciprocal of permeability).
ρ	specific resistance.
ξ	eddy current loss coefficient.
Φ, ϕ	the total magnetic flux in a circuit.
Ψ	induction factor.
χ	ampère-turns.
Ω, ω	angular velocity: radians per second.

ABBREVIATIONS.

Ampères	A.	Watt-hour	w.h.
Volts	V.	metres	m.
Electro-motive force	E.M.F.	centimetres	cm.
Difference of potential	P.D.	millimetres	mm.
Ohms	ω	kilometres	km.
Joules	J.	grammes	g.
Watts	W.	kilogrammes	kg.
Henries	H.		

PREFIXES.

m-	signifying	milli-	$=\times 10^{-3}$
μ	,,	micr- or micro-	$=\times 10^{-6}$
k	,,	kilo-	$=\times 10^{3}$
M	,,	mega- or meg-	$=\times 10^{6}$

Areas and volumes are signified by the indices ^{2} or ^{3} respectively; thus: cm.^{2}, cm.^{3}, etc.

$\odot$ is used to denote the section of a conductor, the current in which is coming up through the paper; + to denote that of a conductor in which the current is going down through paper.

LIST OF FORMULÆ ASSUMED IN THIS BOOK

Units.

1 A. = 10^{-1} c.g.s. unit.
1 V. = 10^{8} c.g.s. units.
1 ω = 10^{9} c.g.s. units.
1 W. = 10^{7} ergs per second.
= 1 joule per second.
= 1/746 H.P.

Electric Circuits.

If V = the P.D. between two points in a conductor carrying a current I, and R = the resistance of the conductor between these two points, $V=IR$.

The specific resistance of a material is the resistance of a conductor made of the material, having a length of 1 cm., and a cross section 1 cm.^{2}

The resistance of a conductor of length l cms., and having a cross section s cm.^{2}. is

$$R=\rho l/s.$$

If R_0 = the resistance at 0°C, and R = the resistance at t°C, then

$$R=R_0\,(1+at).$$

If R_1, R_2, R_3..denote the values of several resistances then their combined resistance, R, is given by

$$R=R_1+R_2+R_3+\ldots\ldots\ldots\text{when they are in series.}$$

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots\ldots\ldots\text{when they are in parallel.}$$

The power wasted as heat, when a current I is sent through a resistance R, is I^2R.

The Magetization of Iron.

If $\mathscr{H}$ denote the strength of the applied magnetic field, and $\mathscr{B}$ =the magnetic induction in the iron, the permeability at that induction being μ, then

$$\mathscr{B}=\mu\mathscr{H} \text{ and } \Phi=\mathscr{B}S.$$

For most engineering materials except iron and steel $\mu=1$.

Magnetic Fields due to Electric Currents.

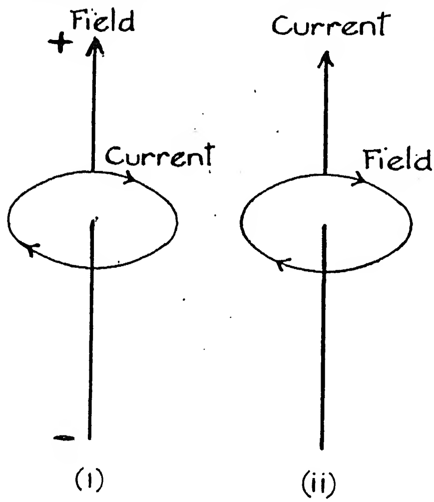


FIG. 1

The laws of directions: (i) In order to push a right-handed corkscrew in the direction of the magnetic field set up by an electric current flowing in a coil, the screw must be turned in the direction of the current; or, if we push a screw along a wire in the direction of the current flowing in the wire, it will turn in the direction of the field due to the current. (See Fig. 1.)

(ii) If a conductor carrying a current be placed in a magnetic field, the mechanical force acting on the conductor

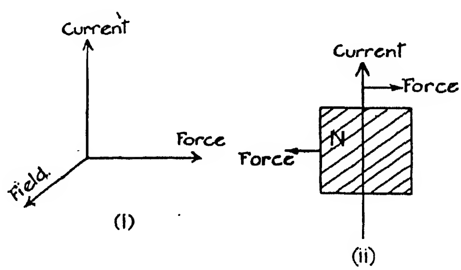


FIG. 2.

is at right angles to the directions of both the current and the field. (See Fig. 2.)

The reluctance of any part of a magnetic circuit is given by

$$\mathscr{R}=\frac{l}{\mu S},$$

the cross-section, material and induction for this part of the circuit being supposed constant.

For any part of a magnetic circuit the drop of magnetic potential is given by $\Phi\frac{l}{\mu S}$; for the whole circuit the total magneto-motive force required is given by

$$\mathscr{F}=\frac{4\pi}{10}NI=\Sigma\frac{l\Phi}{\mu S}.$$

In the central region of a long solenoid or helical coil the magnetic force is given by

$$\mathscr{H}=\frac{4\pi}{10}\frac{IN}{l}$$

where l is the length of the solenoid, and I is the current in ampères.

(iii) The magnetic force at a point P, distant x cms. from the axis of straight conductor carrying a current I ampères, is $\frac{2I}{10x}$, x being measured perpendicular to the axis of the conductor.

CHAPTER I.

INDUCED E.M.F.'s AND CURRENTS.

THE INDUCTION OF CURRENTS IN CIRCUITS.

§ 1. A circuit carrying a current produces a magnetic field in its neighbourhood and therefore a magnetic pole situate in this field will be acted upon by a force. Hence if this pole be moved in the direction of this force, work is done on the pole by the force. Therefore, by the conservation of energy, the work done on the pole must be accompanied by a disappearance of energy in some other part of the field. If the current remains steady, this energy can only come from the source that maintains the current, so that this source must provide more energy than that required to maintain the current when the pole is at rest, the excess being used to supply the work done on the pole. If the battery, or whatever the source may be, supplies energy at a constant rate, part of the energy is used for the work done on the moving pole, so that the current will be less than it would be if the pole were at rest. But the E.M.F. of the battery is constant, and therefore there must be a counter E.M.F. set up, or induced, in the circuit to bring about the reduction of current. This E.M.F. is called the *induced* E.M.F. The magnitude and direction of the induced E.M.F. are given by the following laws :

§ 2. **Faraday's Law.** *Whenever the number of lines of force enclosed by a circuit is changing, there is an* E.M.F. *acting round the circuit, apart from any other source of* E.M.F. *which may be in the circuit. The amount of this induced* E.M.F. *is equal to the rate of* decrease *of the number of lines enclosed by the circuit, i.e.*

$$= -\frac{d\Phi}{dt} \textit{ absolute units.}$$

§ 3. **Lentz's Law.** *The positive direction of the induced* E.M.F. $\left(-\frac{d\Phi}{dt}\right)$ *and the direction in which a line of force must pass through the circuit in order to be counted positive, are related in the same way as the motions of rotation and forward motion of a right-handed screw.*

It is immaterial whether we cause the lines of magnetic force to move across the circuit, or move the circuit across the field of force Faraday's Law will be true in either event; it will also be true if the number of lines cutting the circuit be altered by deforming the circuit or by changing the flux density $\mathscr{B}$

The relation between the direction of the induced E.M.F. and the direction of the magnetic field will be better understood after reading the next article.

§ 3. **Straight Conductor of Constant length moving across a Magnetic Field of Uniform Strength.**

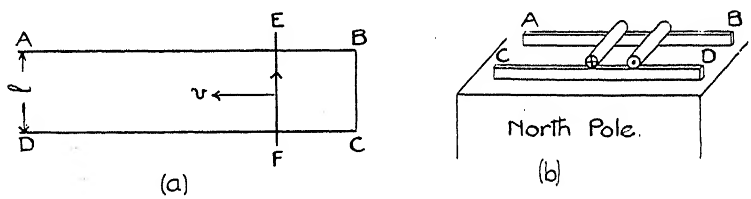


FIG. 3.

Suppose we have two parallel rails AB and CD, l cms. apart, as in Fig. 3, and a conductor EF, lying across them, moving with a velocity v cm./sec. Let $\mathscr{H}$ be the strength of the field, which is supposed to be coming up through the paper, i.e. there is a N-pole underneath the paper. Perhaps the easiest way to see the direction of the induced E.M.F. is as follows: If the conductor had current in it in direction FE (shown by the left conductor in Fig. 3b), there would be a force acting on it from left to right, and it would move in that direction of its own accord. Hence by the principle of conservation of energy, if we wish to move the conductor in such a way that the induced current shall be in the same direction, we shall have to overcome this force, and move the conductor from right to left. The "positive

direction" of the induced E.M.F. in Lentz's Law refers to $\left(-\frac{d\Phi}{dt}\right)$, so that the induced E.M.F. is $-\frac{d\Phi}{dt}$ in direction EF or $\frac{d\Phi}{dt}$ in direction FE, and obviously $\frac{d\Phi}{dt} = \mathcal{H} lv$.

$$\therefore \text{ induced E.M.F.} = \mathcal{H} lv \text{ in direction FE (c.g.s. units)}$$
$$= \mathcal{H} lv 10^{-8} \text{ volts.}$$

If I (c.g.s. units) be the current flowing, work is being done at the rate of $I\mathcal{H} lv 10^{-7}$ watts. There will be a force between EF and the N-pole; this force tries to make the conductor move to the right, and we have to apply a force, F dynes say, if we wish EF to move to the left. Then the work done by $F = Fv$ ergs/sec.,

$$= Fv 10^{-7} \text{ joules per sec.}$$

Hence we have $\quad Fv10^{-7} = I\mathcal{H} lv 10^{-7}$,

$$\therefore \quad F = I\mathcal{H} l \text{ dynes} \qquad \text{(1).}$$

This is, therefore, the force acting on the conductor, when it carries a current I across a field $\mathcal{H}$. If the conductor is free this is the force which will give it motion one way, or the force we must apply to make it move the other way.

§ 4. **Flux through a Coil.** If we have a flux Φ passing through the space enclosed by a single turn of wire, the E.M.F. induced is $-\frac{d\Phi}{dt}10^{-8}$ volts. Hence, if we have a coil of N turns, i.e. N single turns in series, the total E.M.F. will be

$$-N\frac{d\Phi}{dt}10^{-8}$$
$$= -\frac{d}{dt}\left(\Phi N\right)10^{-8} \text{ volts} \qquad \text{(2).}$$

Hence we must always deal with flux-turns, and not flux alone. Some writers use the phrase "linkages," and some loosely write flux, when they mean flux-turns.

§ 5. **Example.** A closed coil of wire has 1000 turns, and its resistance is 40 ohms. 14000 magnetic lines are introduced into the coil by thrusting a magnet into it, the whole operation taking 0·2 second. What is the average induced E.M.F., and what is the average current? (Special Exam. in Applied Sciences, Cambridge, 1908.)

The average rate of change of flux $= \dfrac{14{,}000}{0{\cdot}2}$

$= 70{,}000$ lines per sec.

$\therefore$ the average E.M.F. induced $= 70{,}000 \times 1000,$

$= 0{\cdot}7.10^8$ absolute units,

$= 0{\cdot}7$ volts.

the average current $= \dfrac{0{\cdot}7}{40} = 0{\cdot}0175$ amps.

SELF-INDUCTION.

§ 6. If, for any reason, the current in a circuit changes, the flux set up by the current, and therefore the number of lines enclosed by the circuit, will also change. The change of flux will induce an E.M.F. in the circuit, and, by the conservation of energy, its direction must be such as to oppose the change of flux. This induced E.M.F. is called the **E.M.F. of Self-Induction.**

First consider the case of a long solenoid, or a helix, we have, if ϕ=the flux through the coil at any instant, and N=the number of turns in the coil,

$$\phi = \mathscr{B} S = \mu \mathscr{H} S = \mu \frac{4\pi}{10} \frac{i N}{l} S$$

where i is the value of the current at the instant under consideration.

The self-induced E.M.F. will be

$$e = - N \frac{d\phi}{dt} 10^{-8} \text{ volts}$$

If μ is constant we can write this:

$$e = - \frac{4\pi\mu S N^2}{l} \frac{di}{dt} 10^{-9}$$

$$= - L \frac{di}{dt} \ldots \text{volts}, \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\text{where } L = \frac{4\pi\mu S N^2}{l} 10^{-9} = \frac{N\phi}{i} 10^{-8}, \ldots\ldots\ldots\ldots (4)$$

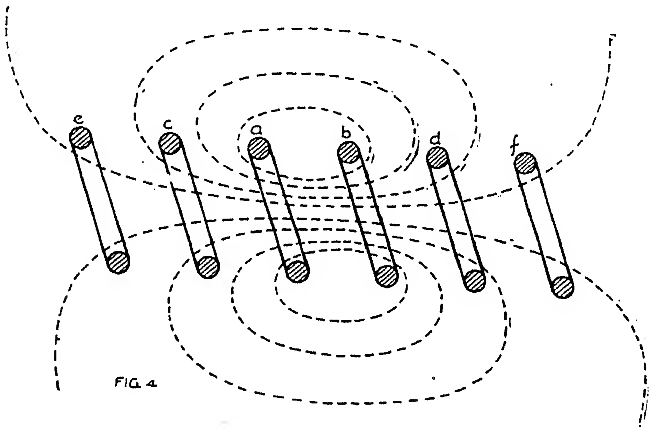
ϕ being the value of the flux when the current in the coil is i.

L is called the **Coefficient of Self-Induction,** or the **Self-Inductance,** of the coil, and, when expressed as above, is measured in *Henries*, on the volt-ampère system of units.

Since I volt $=10^8$ c.g.s. units, and I amp. is 10^{-1} c.g.s. unit, we see that an Henry is 10^9 c.g.s. units of self-induction.

When iron is included in the circuit, L is not constant, since μ depends on ϕ, so that the value of L will vary with i, and the best definition of L is : the coefficient of self-induction of a coil, when carrying a current I ampères, is $10^{-8}\times$ the number of flux turns set up by that current, divided by the value of the current ; the result so found is in Henries.

In the case of a short coil, the above definition still obtains, but the numbers of lines of flux passing through each turn must be taken separately and added together.



Thus, in the case shown in Fig. 4, which is supposed to represent the flux due to I amp.,

the two turns,		*a* and *b*,		are threaded by		8 lines	
,,	,,	*c* ,,	*d*	,,	,,	4	,,
,,	,,	*e* ,,	*f*	,,	,,	2	,,

Hence we have

$$L = (2\times 8+2\times 4+2\times 2)\ 10^{-8}$$
$$= 28.\ 10^{-8} \text{ Henries.}$$

§ 7. **E.M.F. of Self-Induction.** The E.M.F. of self-induction has been expressed, above, as

$$e = -L\frac{di}{dt},$$

but, taking the definition of L given on p. 4, this equation

is not true if L is variable, as in the case of circuits containing iron. In this case we must write

$$e = -\frac{d}{dt}(Li) = -\left(L\frac{di}{dt} + i\frac{dL}{dt}\right)$$

$$= -\frac{4\pi SN^2}{l}10^{-9}\left(\mu\frac{di}{dt} + i\frac{d\mu}{dt}\right) \quad ..(5).$$

§ 8. **Closing and Opening a Circuit possessing Self-Inductance, assuming a Constant L.**

Let r = the resistance of the circuit,
L = coeff. of self-induction
V = applied P.D., when the switch is closed.

The induced E.M.F. $= -L\dfrac{di}{dt}$.

This opposes the flow of current, and a pressure equal and opposite to this must be applied, apart from that required for resistance. Hence we must have, when the switch is closed

$$V = L\frac{di}{dt} + ir \dots\dots\dots\dots\dots\dots (6)$$

This may be written $V - ir = -\dfrac{L}{r}\dfrac{d}{dt}(V - ir)$.

$$\therefore \quad V = ir + A\epsilon^{-\frac{rt}{L}}$$

where A is a constant to be determined.

Now, when $t = 0$, $i = 0$, $\therefore A = V$.

This leads to

$$i = \frac{V}{r}\left(1 - \epsilon^{-\frac{rt}{L}}\right) \dots\dots\dots\dots\dots\dots (7).$$

When we break the circuit we have

$$0 = L\frac{di}{dt} + ir,$$

which gives $\qquad i = A\,\epsilon^{-\frac{rt}{L}}.$

If $i = I$ before the switch is opened, i.e. when $t = 0$, we have, then,

$$i = I\,\epsilon^{-\frac{rt}{L}}$$

It is self-induction which is the cause of arcing when a circuit possessed of this quality is suddenly broken; if the E.M.F. of self-induction be great enough to overcome the resistance of the air-gap caused by the break, the current will be kept flowing and 'arc' across the gap. We shall return to this matter when dealing with the commutation of dynamos and motors.

§ 9. The graph of i as a function of t, when the switch is closed, is shown in Fig. 5, from which it will be seen that i rises gradually to its value V/R, given by Ohm's Law. The time taken to approach this value is small if L is small, and large if L is large.

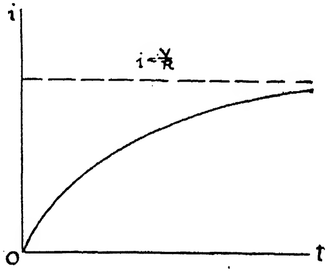


FIG. 5

§ 10. The physical explanation of the fact that i rises gradually, and not instantly, is easily seen: at any time the rate of working against self-induction is $Li\dfrac{di}{dt}$.

$$\therefore \quad \text{the work done in time } dt = Li\frac{di}{dt}\,dt.$$

$$\therefore \quad \text{,, ,, ,, ,, ,,} \quad t = L\int_0^t i\frac{di}{dt}\,dt,$$

or
$$W = \tfrac{1}{2}Li^2, \qquad \ldots\ldots\ldots\ldots\ldots\ldots (9).$$

and this amount of energy has to be supplied by the electric forces before i attains its final value. It is stored in the form of potential energy; when the circuit is opened this energy is given up, and tends to maintain the current, the value of which falls gradually to zero.

§ 11. **Example 1.** A coil consists of 1200 turns wound on a cylinder of non-magnetic material, and is such that the total flux through each turn produced by a current of 1A is $1\cdot5.\ 10^6$. Determine the coefficient of self-induction of the coil measured in Henries. (Mech. Sc. Trip. 1911.)

$$L = \frac{1200 \times 1\cdot5 \times 10^6}{1} \times 10^{-8} \quad = 18 \text{ Henries.}$$

Example 2. An iron ring has a mean length of 200 cms., a sectional area of 100 cm²., and is wound with 2500 turns of wire. Taking $\mu=1200$ when the current is 1 amp., calculate L for the coil,

We have

$$\mathscr{H}=\frac{4\pi}{10}\cdot\frac{1\times 2500}{200}=5\,\pi.$$

$$\mathscr{B}=\mu\,\mathscr{H}=5\times 1200\times\pi.$$

$$\Phi=\mathscr{B}\,S=6000\,\pi\times 100.$$

$$=6\,\pi\times 10^5.$$

$$L=\frac{N\Phi}{i}10^{-8}=2500\times 6\pi\times 10^5\times 10^{-8}.$$

$$=47 \text{ Henries.}$$

Example 3. The coil described in Ex. 1, p. 7, has a current of 5 amps. flowing in it; calculate the energy of the coil due to self-induction, measured in ft. lbs. (Mech. Sc. Trip. 1911.)

On p. 7, we found $L=18$ Henries.

Hence, by equation (9) the energy stored is:

$$W=\tfrac{1}{2}Li^2 \text{ joules,}$$

$$=\tfrac{1}{2}\times 18\times 25 = 225 \text{ joules.}$$

$$=225\times 0{\cdot}737 \text{ ft. lbs.,}$$

$$=166 \text{ ft. lbs.}$$

Example 4. A coil whose self-inductance is 0·15H., and resistance 0·6ω., is suddenly connected to a source of constant E.M.F. Find how long elapses before the current reaches 90 per cent. of its full value.

According to equation (7), the current is given by

$$i=\frac{V}{r}\left(1-\epsilon^{-\frac{rt}{L}}\right)=\frac{V}{0{\cdot}6}\left(1-\epsilon^{-\frac{0\cdot 6}{0\cdot 15}t}\right).$$

Therefore we require

$$1-\epsilon^{-4t}=0.9,$$

or

$$\epsilon^{-4t}=0.1,$$

which gives

$$t=0{\cdot}576 \text{ sec.}$$

§ 12. **Self-Inductance of a Pair of Parallel Wires.** We

seek the self-inductance of two long parallel wires, A and B, one out, and one return; for example, the cables of an overhead transmission line.

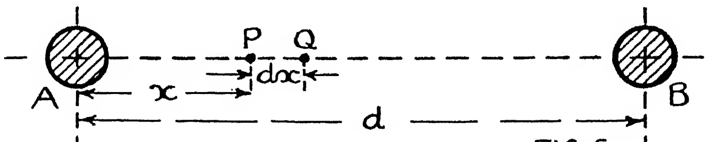


FIG. 6.

Let r = the radius of the section of the wires, and I amps. be the current flowing.

The magnetic force at P, due to A, is $\frac{2I}{10x}$.

$\therefore$ the induction at P $= \frac{2I}{10x}$, since $\mu = 1$ for air.

and the flux through PQ $= \frac{2I\,dx}{10x}$,

per centimetre of double wire.

$\therefore$ the total flux through the space between the surface of A and the centre of B, due to A,

$$= \int_r^{d-r} \frac{2I}{10x} dx + \int_{d-r}^{d} \frac{2I}{10x} dx$$

$$= \frac{2I}{10}\left(\log_e \frac{d}{r} + \log_e \frac{d}{d-r}\right) = \frac{2I}{10} \log_e \frac{d}{r}.$$

For the two wires it will $= \frac{4I}{10} \log_e \frac{d}{r}$ per cm. of double wire.

If in each wire we neglect the flux due to the current in that wire, the value of L, per cm. of line, is

$$L = 10^{-8} \frac{4}{10} \log_e \frac{d}{r},$$

$$= 4 \times 10^{-9} \log_e \frac{d}{r} \quad \text{Henries.... (10)},$$

which is an approximate expression for L, sufficiently accurate, except in cases when the wires are close together; it then becomes necessary to consider the flux cutting the wires themselves.

Consider a point P, Fig. 7, inside conductor A, at a distance a from the centre.

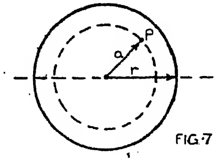


FIG. 7

The current density in the wire is $\dfrac{I}{\pi r^2}$.

$\therefore$ the amount of current within the circle through P, assuming uniform distribution, is

$$\frac{I}{\pi r^2} \times \pi a^2 = I\frac{a^2}{r^2}.$$

The magnetic force at P is the same as if all this were concentrated at the centre, since the current outside the circle through P has no effect.

$\therefore$ the magnetic force at P $= \dfrac{2I}{10a}\dfrac{a^2}{r^2} = \dfrac{2Ia}{10r^2}$.

$\therefore$ the induction at P $= \dfrac{2Ia}{10r^2}$.

$\therefore$ flux at P, through da per cm. length, $= \dfrac{2Ia\,da}{10r^2}$.

Now, the self-inductance is equal to the number of magnetic linkages, when the current is one ampère, multiplied by 10^{-8}. Hence, for the flux within one wire, the contribution to L is

$$\int_0^r \frac{2a\,da}{10r^2} \times 10^{-8}$$

$$= 10^{-9} \quad \text{Henries.}$$

Hence, for both wires the total value of L is

$$\left(4 \log_e \frac{d}{r} + 2\right) 10^{-9} \quad \text{Henries.... } \quad (11)$$

When the distance between the wires is large compared to their radii, the second term is negligible.

§ 13. **Example.** Two parallel straight conductors, each 2 mm. in diameter, are four metres between their centres, and carry the same current flowing in opposite directions. Neglect the flux which cuts the conductors, and find the coefficient of self-induction for one kilometre length of double wire. (Mech. Sc. Trip. 1911.)

Here $d = 400$ cm.

$r = 0 \cdot 2$ cm.

$$L = 4 \times 10^{-9} \log_e \frac{400}{0 \cdot 2} \quad \text{Henries per cm.}$$

$$= 4 \times 10^{-9} \log_e 2000 \times 10^5 \text{ Henries per kilometre,}$$

$$= 27 \cdot 76 \times 10^{-4},$$

$$= 0 \cdot 00276\text{H., per kilometre of double wire.}$$

EDDY CURRENTS.

§ 14. Iron, being not only a good magnetic conductor, but also an electric conductor, when set rotating in a magnetic field, or when subjected to an alternating flux, allows the passage of electric currents due to the E.M.F. induced in the iron. For example, consider the case of a solid cylinder of iron rotating about its axis in a magnetic field, the latter being in

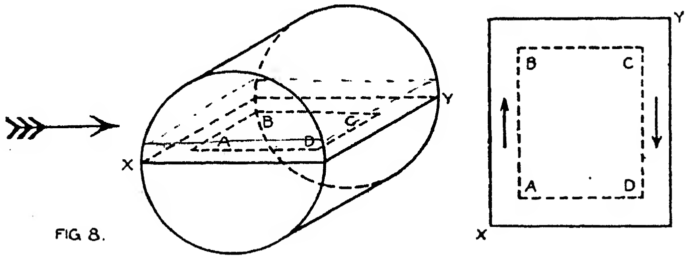


FIG 8.

the direction of the large arrow. (Fig. 8.) Consider the plane XY, shown separately on the right of the diagram: A strip such as AB will have an E.M.F. induced in the direction shown, whilst CD will have one in the opposite direction; there will be no E.M.F.'s induced in the directions AD and BC. Hence a current will circulate approximately in the path ABCDA. Such currents are called *eddy currents.*

If E be the E.M.F. induced in an eddy current path of resistance R the loss of power is E^2/R. In the case of the solid cylinder the resistance to eddy currents is very small, and consequently the loss of power would be very great, but if the cylinder be built of a number of insulated discs the resistance of the eddy current paths is greatly increased. and consequently the power wasted is also reduced without materially increasing the reluctance of the magnetic path. For this reason the armatures of D.C. machines, and the iron

parts of all alternating current apparatus, are built of thin insulated stampings instead of solid castings.

§ 15. **Power Consumed by Eddy Currents.**

Let $\mathscr{B}$ = the flux density of the field causing the eddy currents.

f = the number of magnetic cycles per second.

Then the E.M.F. induced is proportional to $\mathscr{B}f$.

$\therefore$ the corresponding current is proportional to $\frac{\mathscr{B}f}{\rho}$, where ρ is the sp. resistance of the metal.

Hence the power consumed is proportional to $\frac{\mathscr{B}f}{\rho} \times \mathscr{B}f$, *i.e.* to $\frac{\mathscr{B}^2 f^2}{\rho}$, or, taking into account the change of temperature produced, we can write:

Loss of power due to eddy currents $\propto \frac{\mathscr{B}^2 f^2}{\rho(1+at)}$ ....(12)

where t is the temperature in °C.

§ 16. **Calculation of Eddy Current Losses.** In most cases we cannot calculate with exactness the loss due to eddy currents, as we cannot predict exactly the variations of flux density in the material, or determine the resistances of the paths. But if we assume that the metal is sufficiently subdivided for us to take the induction as constant over a cross section of one part, we can predict with sufficient accuracy the losses in two particular cases: wires, and plates of rectangular cross section.

Eddy Currents in Cylindrical Wires. Assume that the lines of magnetic induction are parallel to the axis of the wire, then, when the induction varies, cylindrical currents will be set up in the wire. Consider one of these currents, having a radius r, and suppose that the magnetism is a simple periodic function of the time, so that we can write

$$\mathscr{B} = \mathscr{B}_m \sin 2\pi ft$$

where f is the number of magnetic cycles per second, and $\mathscr{B}_m$ is the maximum value of $\mathscr{B}$. We have, then,

the total flux within the circle of radius $r = \pi r^2 \mathscr{B}$.

$\therefore$ instantaneous E.M.F. induced is

$$e = -\pi r^2 \frac{d\mathcal{B}}{dt} 10^{-8}$$
$$= -\pi r^2 \quad 2\pi f. \quad \mathcal{B}_m \cos 2\pi ft.\ 10^{-8}$$
$$= -2\pi^2 r^2 f 10^{-8}.\ \mathcal{B}_m \cos 2\pi ft.$$

Consider 1 cm. of the wire. The resistance of the path, of thickness dr, is

$$\frac{\rho \times 2\pi r}{dr}.$$

The power wasted, in the element, is

$$\frac{e^2 . dr}{2\pi r \rho}$$
$$= \frac{2\pi^3 r^3 f^2 \mathcal{B}^2_{max} \quad \cos^2 2\pi ft}{\rho \cdot 10^{16}} dr.$$

To obtain the total power wasted, we must take the mean of this during one period of the flux, and integrate over the cross section. This gives

$$P = \frac{2\pi^3 f^2 \mathcal{B}^2_{max}}{\rho \cdot 10^{16}} \int_0^R f \int_0^{\frac{1}{f}} r^3 \cos^2 2\,\pi ft.\, dr\, dt.$$

$$= \frac{2\pi^3 f^2 \mathcal{B}^2_{max}}{\rho . 10^{16}} \int_0^R \frac{r^3\, dr}{2}$$

$$= \frac{\pi^3 f^2 \mathcal{B}^2_{max} R^4}{4\rho . 10^{16}}$$

Watts per cm. of wire. . . . (13)

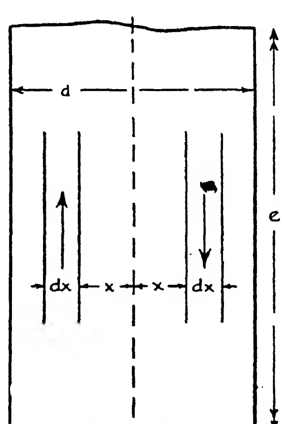


FIG. 10

We see that this is proportional to the fourth power of the diameter.

§ 17. **Eddy Currents in Sheets of Rectangular Cross Section** (e.g. laminated iron).

Let b = thickness of plate. Consider two slices, of thickness dx, at distances x from the middle plane; if the disis plate be so thin that the ends connecting these slices may be neglected,

then the latter may be regarded as composing an elementary eddy-current path.

Let l=length of sheet, in the plane of the paper, and perpendicular to x. As before, let $\mathcal{B}=\mathcal{B}_m \sin 2\pi ft$.

Then
$$e=-\frac{d}{dt}(\mathcal{B}\ 2x.l)10^{-8}$$
$$=-4\pi fxl10^{-8}\ \mathcal{B}m \cos 2\pi ft.$$

The resistance of the path, 1 cm. deep, perpendicular to the plane of the paper, is
$$\rho\frac{2l}{bx}.$$

Thus, the loss of power in the whole piece, 1 cm. deep, is
$$P=\frac{8\pi^2 f^2 l.\mathcal{B}^2{}_m}{\rho.10^{16}}\int_0^{\frac{b}{2}} f\int_0^{\frac{1}{f}} x^2 \cos^2 2\pi ft.\ dx.\ dt$$
$$=\frac{\pi^2 f^2 \mathcal{B}^2{}_m\ .\ b^3 l}{6\rho 10^{16}} \quad \text{.......................} \quad (14).$$

Hence the loss per c.c. is $\frac{\pi^2 f^2 \mathcal{B}^2{}_m b^2}{6\rho 10^{16}}$ watts,

and this varies as the square of the thickness of the plates.

§ 18 **Example.** A copper disc at a temperature of 20°C. rotates in a magnetic field and requires a torque of 5 lbs. ft. to maintain rotation. If the speed and field strength be both doubled, find the couple then required, if the temperature rises to 50°C., and if the temperature coefficient of copper be 0·004 per degree Centigrade. (Inter-collegiate Exam., Cambridge, 1910.)

Let M_1=the torque in the first case, and M_2 the torque in the second case. Then, since the power spent is proportional to $\frac{\mathcal{B}^2 f^2}{\rho}$, we can write
$$M_1=\frac{\lambda f\mathcal{B}^2}{R_0(1+0{\cdot}08)}=\frac{\lambda f\mathcal{B}^2}{1{\cdot}08R_0}$$

and
$$M_2=\frac{\lambda.2f(2\mathcal{B})^2}{R_0(1+0{\cdot}2)}=\frac{8\lambda f\mathcal{B}^2}{1{\cdot}2R_0}$$

where λ is a constant.

Then, since $M_1=5$, we have

$$M_2=\frac{8\times 1\cdot 08\times 5}{1\cdot 2}=36\text{ lbs. ft.}$$

EXAMPLES.

1. A magnet has its north pole a cylinder 5 cm. in diameter and 10 cm. high; its south pole is a concentric cylinder. The flux density just outside the N. pole is 2000 lines per cm^2. A circular coil of wire of 100 turns has its axis coincident with that of the N. pole. It drops from top to bottom of the cylinder in $\frac{1}{10}$ second. Find the average E.M.F. produced. If the resistance of the circuit containing the coil is 50 ohms, find the number of coulombs discharged during the fall.

2. Calculate the coefficient of self-induction of a coil of 2,000 turns wound on a wooden core, having a flux of 10^5 lines per ampère per turn.

3. A ring of iron is wound with 600 turns, its mean radius is 10 cm., and it is excited by a current of 5 amps. The permeability of the iron is such that when $\mathscr{H}=60$, $\mathscr{B}=16{,}000$. Calculate the self-inductance of the coil.

If the iron has an air-gap, 0·1 cm. wide cut in it, find the new value of L, assuming the $\mathscr{H}$-$\mathscr{B}$ curve to be straight from the given point to the point $\mathscr{H}=30$, $\mathscr{B}=14{,}400$.

4. A short coil of 10 turns is wound on a core of non-magnetic material. The current flowing is one ampère, and the distribution of flux is such that the number of lines of force between each turn is as shown (cf. Fig. 11).

Calculate the value of L.

5. The coil of Ex. 1, p. 7, is joined in series with a non-inductive resistance of 20 ohms, and the combination is subjected to a steady pressure of 500 volts, which causes a current of 10 amps. to flow. If the coil be suddenly shorted by a copper bar of negligible resistance and self-inductance, find how much current remains in the coil 1 second after contact is made. (Mech. Sc. Trip. 1911.)

6. A pair of solid conductors of 0·5″ diameter run a distance of 10 miles, and are 24″ apart. Find the coefficient of self-induction of the line, measured in Henries. What is the energy stored when a current of 500 amps. is flowing?

7. A cable is formed of two concentric conductors of equal cross section, the inner one being solid, and of radius r, whilst the inner radius of the outer conductor is R. Shew that the coefficient of self-induction, per cm., is

$$\left[2\log_e \frac{R}{r} + \left(\frac{R^2+r^2}{r^2}\right)^2 \log_e\left(\frac{R^2+r^2}{R^2}\right) - \frac{R^2+r^2}{r^2}\right] 10^{-9} \text{ Henries.}$$

CHAPTER II.

DIRECT-CURRENT GENERATORS.

THE PRINCIPLE OF THE DIRECT-CURRENT GENERATOR.

§ 19. It was shown on p. 2 that if we move a conductor across a magnetic field, an E.M.F. will be induced in the conductor; this is the underlying fundamental principle of the generator. The most obvious way to effect this is to mount the conductor on the circumference of a cylinder, and rotate the latter between the poles of a magnet, and we may as well put conductors all round the cylinder, for then we shall get an E.M.F. induced in all of them. We must then connect the ends of the conductors together in such a way that the several E.M.F.'s add up, and provide an arrangement for collecting the current. The cylinder on which the conductors are mounted is called the *armature*.

§ 20. A simple generator would consist of a wooden armature rotating between the poles of a permanent magnet, but such a machine would not be very efficient. The first alteration to make is to have an iron armature so that the path for the magnetic flux is made as easy as possible. Such a machine working with a permanent magnet is called a magneto, and is chiefly used for ignition with internal combustion engines. For ordinary purposes we do away with the permanent magnet, and provide an electromagnet, using the current from the armature to provide the magnetizing force; the chief reason for this is the difficulty of making large magnets retain their magnetism, particularly in face of armature reaction. The magnet between the poles of which the armature revolves is called the *Field Magnet*. Instead of having only one pair of poles, we can have poles all round the armature, and then the machine is then said to be *multipolar*.

ARMATURES AND COMMUTATORS.

§ 21. Let us now see how we are going to arrange the conductors and collect the current. One way is to make our cylinder hollow, and wind a coil of wire continuously round the annulus, as shown in Fig. 12 (i). Such an armature is called a ring armature. With the N. pole of the field magnet on the left, and direction of rotation of the armature anti-clockwise, the directions of the induced currents will be as shown. All the E.M.F.'s on one side add up, and all those on the other also, while at A and B the two E.M.F.'s, and therefore the currents, are in opposition, so this is the place to collect the current. If we allow two flat bars of copper to bear on the armature conductors here, the currents due to the two E.M.F.'s will flow through these strips, flowing into the strip at A, and, if the circuit outside be closed, back into the conductors by B.

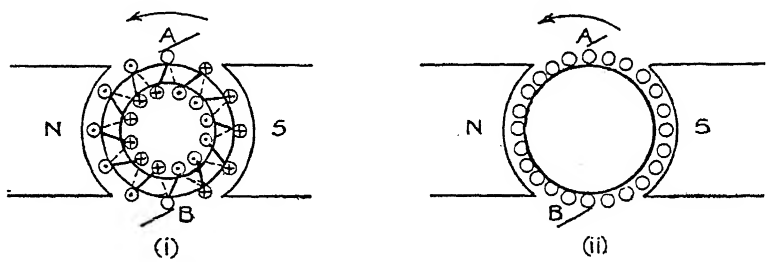


FIG. 12

§ 22. **Disadvantages of the Ring Armature.** In the first place the iron path for the flux is not so good as if the cylinder, or core, were solid. In the second place those conductors inside the core are cutting practically no flux, and so are useless; and, moreover, they are worse than useless, for they do cut some flux, but in such a direction that the E.M.F.'s induced are opposed to those induced in the outside conductors.

Now it will be seen that every conductor on the inside, on the left say, has the current flowing in the same direction as the outside conductors on the opposite side, and so let us shift these inside conductors to a point immediately opposite on the outside, as in Fig. 12 (ii), and then all are useful, and we get approximately twice as much E.M.F. for the same axial length of conductors, but the end connections are larger. Such an armature is called a *Drum Armature.*

§ 23. **Commutators.** In reality the copper strips do not bear upon the armature conductors; if they did, when the conductors became worn the whole winding would have to be replaced, and its resistance would be increased by the reduction of sectional area. Instead, a separate contrivance, called a commutator, is employed. The commutator consists of a number of heavy copper bars, which are mounted on the surface of a cylinder on the same shaft as the armature. The copper bars are parallel to the axis of the shaft, are insulated from each other, and from the cylinder to which they are attached. These bars are connected to the armature winding, and the brushes rest upon them, instead of bearing upon the armature wires. The connections between the armature conductors and the commutator segments, as they are called, will be understood when we have studied armature windings in detail.

ARMATURE WINDINGS.

§ 24. The winding of an armature may be regarded as consisting of a number of copper bars or wires, carried in slots in the periphery of the core, and connected together at the ends in such a manner as to form a number of endless closed circuits. If all the bars are connected into one endless circuit, the winding is said to be *singly re-entrant*, but if the winding is divided into several independent endless circuits it is called *multiply re-entrant*.

Such an armature cannot have less than *two* paths in parallel from brush to brush, as will readily be seen from any of the diagrams given here. The current coming in at one brush divides equally between the two paths, uniting again at the other brush.

§ 25. **Pitch.** The pitch of a winding is the number of conductors by which we advance with each end connection. Thus, if we are at No. 2, and the pitch be 9, the end connection from No. 2 will take us to No. 11, and so on.

The forward pitch is the pitch at the end remote from the commutator, whilst the backward pitch refers to the commutator end of the winding. These are not necessarily the same.

In the diagrams here given there are two views shewn: One shows the armature viewed from the end, whilst in the

other the winding is supposed to be cut in two and laid out flat on the table. The square shaded areas represent the pole faces.

§ 26. **Multipolar Windings** are classed under two main heads :

(1) *Lap*, or *Multiple-Circuit* windings.
(2) *Wave*, or *Two-Circuit* windings.

The geometrical distinguishing feature is that in Lap-Winding we go forwards at the end remote from the commutator, and backward at the commutator end, or vice versa. Thus, in Fig. 13, we go forwards from (1) to (6) at the remote end, and

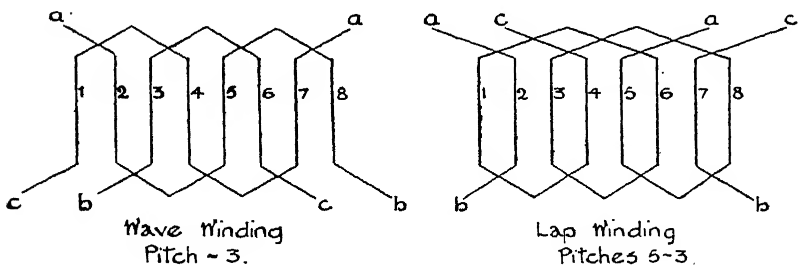


FIG. 13.

back to (3) at the near end. In a wave winding, however, we go forwards all the time, thus (Fig. 13) we connect (1) to (4), then (4) to (7), (7) to (2), and so on.

In a lap winding, if the forward pitch be greater than the backward pitch, the winding as a whole carries us gradually forwards round the armature, and is called *progressive*. If the forward pitch be less than the backward pitch, we gradually work backwards round the armature as we follow the winding, which is then called *retrogressive*.

Wave winding is progressive or retrogressive according to whether, in going round the armature, on the first return to the polar space from which we started, we arrive in front of or behind the conductor from which we set out.

Electrically the difference between the two windings is that, with lap winding, there are as many parallel paths as the machine has poles, and as many brushes, whereas a wave-wound armature can only have two paths in parallel whatever be the number of poles ; for each independent winding on the

armature, in both cases. The best way to understand this is to follow carefully the annexed diagrams.

§ 27. The first diagram in Fig. 14 shows an ordinary lap winding with 24 surface conductors, the forward pitch being 7, and the backward 5. It will be seen that the path of the current is like this:—

$$+\text{ terminal}\left\{\begin{array}{l}\text{brush } 2\left\{\begin{array}{l}5-12-7-14\\8-1-6-23\end{array}\right\}\text{brush } 3\\ \text{brush } 4\left\{\begin{array}{l}17-29-19-2\\10-13-18-11\end{array}\right\}\text{brush } 1\end{array}\right\}-\text{ terminal}$$

There are four poles, four brushes, and four parallel circuits through the armature.

This winding is called *Simplex*, and as it closes on itself once after taking in all the conductors, it is called *Singly Re-entrant*.

§ 28. The next diagram shows two such windings, one shown thick and one thin, the thick and thin conductors being placed alternately. This is called *Duplex*, and as we have two windings, each closing on itself, it is called *Doubly Re-entrant*. There are four brushes, and eight parallel circuits. Similarly, we can have Triplex Triply Re-entrant windings and so on. If there are m windings and $2p$ poles, there will be $2mp$ parallel circuits.

§ 28. The third picture shows two windings, but they are not quite independent. The end of the thick winding joins the beginning of the thin, and the end of the thin joins the beginning of the thick, so that the whole winding only closes on itself once. This is a *Duplex Singly Re-entrant* winding. Here again there are eight paths through the armature.

Note that two conductors which are electrically adjacent (e.g. 1 and 8 in the top of Fig. 14) have the current in them flowing in different directions along the armature, and so must be under unlike poles, and it is usual to place them approximately under corresponding parts of their respective poles, so that the forward pitch is generally about equal to

$$\frac{\text{number of conductors}}{\text{number of poles}}.$$

§ 29. Fig. 15 shows a similar series of wave windings. Consider the first diagram. The E.M.F.'s oppose each other

at the brushes 1 and 2, and so brushes must be placed there. These brushes divide the winding into two paths, each of which uses up half the conductors. The same thing happens if we have the brushes 3 and 4. Either pair may be used, so why not use both? We can, and do; brushes 1 and 3 may be connected in parallel, also 2 and 4. The conductors in series between 1 and 3 are all approximately at neutral points, and so at the same potential. If this were not so we might have the same difference of potential between 1 and 3 as between 2 and 4, whilst 1 and 3 were themselves at different potentials. Electrically there is no difference whether we use 2 brushes or $2p$ brushes; the advantage is that we have a bigger contact area for the same commutator dimensions. In the duplex wave winding we have four paths, in the triplex six, and so on, irrespective of the number of poles.

§ 30. **Rules.*** The total number of conductors must be even.

The pitches for a singly re-entrant winding must be odd, and in lap windings their difference must be twice the number of windings in all cases.

The forward pitch will generally be approximately equal to the number of conductors ÷ number of poles.

In singly re-entrant windings half the number of conductors (i.e. the number of coils) must not be divisible by the number of windings.

In lap windings the re-entrancy is the greatest common measure of half the number of conductors and the number of windings; in wave windings it is the G.C.M. of the number of conductors and the pitch.

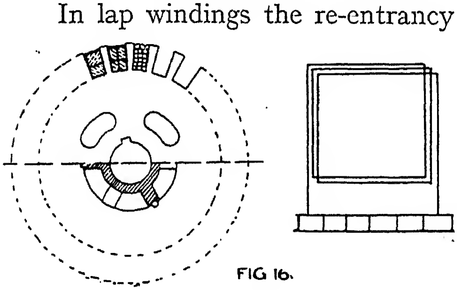


FIG 16.

§ 31. So far we have supposed that there is only one turn round the armature for each commutator section; in practice this

* For further details of armature windings, the reader is referred to the standard text books of dynamo design, such as Cramp's *Continuous Current Machine Design*. The subject is one of great complexity, and space does not permit us to give anything like a complete treatment.

supposition is, in general, untrue. The conductors lie in slots or channels along the periphery of the armature, as shown in section in the left of Fig. 16. In large machines and in machines designed for a low voltage the conductors consist of heavy copper bars, as shown in the two slots to the left, having one and two conductors per slot respectively. In smaller machines and in high voltage machines the armature is usually coil-wound, as shown on the right of Fig. 16. Suppose we begin winding at slot 1 and have to go to slot 4, say: we start at the commutator section corresponding to 1, pass the wire along slot 1, round the end of the armature to 4, back through 1 and so on, round and round until the coil is complete, when we connect the end to the proper commutator section.

EXPRESSIONS FOR THE E.M.F.

§ 32. Suppose we have a coil AB, rotating about O in a magnetic field as shown in Fig. 17; as the coil rotates the number of lines cutting the coil will continually vary, and consequently there will be an E.M.F. induced in the coil.

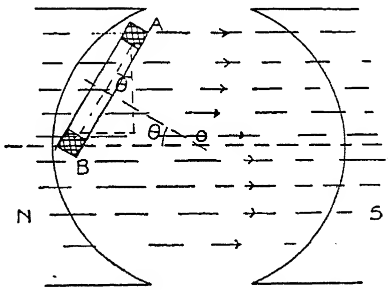


FIG. 17.

Now, let ϕ be the instantaneous value of the flux through the space enclosed by the coil, and Φ the maximum value. When the plane of the coil is at right angles to the field the flux through it is a maximum, and it becomes zero when the coil is parallel to the field. This change happens four times in one revolution. Hence if the coil has N turns in series, and make n revolutions per second, the average rate of change of flux-turns, i.e. the *average* E.M.F. generated, is

$$4\Phi N n 10^{-8} \text{ volts} \qquad \ldots\ldots (15).$$

Also, we have $\qquad \phi = \Phi \cos\theta.$

$\therefore$ if e be the instantaneous E.M.F. and E_m the maximum.

$$e = -\frac{d}{dt}(\phi N) = \Phi N \sin\theta . \frac{d\theta}{dt}$$
$$= \Phi N \omega . \sin\theta = 2\pi n \Phi N \sin\theta .$$
$$= E_{\mathrm{m}} \sin\theta . \quad \quad (16).$$

§ 33. Instead of regarding the armature winding as a number of conductors connected together at the ends, we may consider it as a number of coils, connected together and rotating about an axis parallel to their own planes, namely, the axis of the armature. The E.M.F. in each of these coils undergoes the same periodic changes, separated by a common interval of time. Consider Fig. 18: here all the individual curves of instantaneous E.M.F., in the several coils which are in series between a pair of brushes, are plotted on a common time base. Each coil is separated, in space, by a fixed interval from the next succeeding coil in series, and, since the armature surface describes equal intervals of space in equal intervals of time, the curves of Fig. 18 will all have the same shapes and sizes, separatedby equal intervals of time.

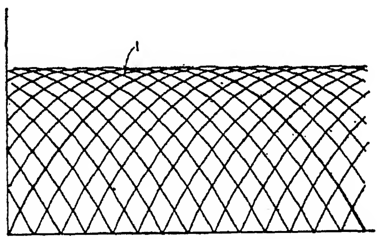


FIG 18

Since the coils are in series, the total E.M.F. between the brushes, at any instant, will be the sum of the ordinates of the several curves at the single point representing the instant under consideration. The same ordinates, however, would be obtained by taking them equally spaced along one of the curves, the spacing being equal to that of the curves themselves, and the number the same as the number of coils in series. The sum of such a system of ordinates is the number of ordinates multiplied by the mean height of the curve over a half-period, and is constant.

Hence the total E.M.F. between a pair of brushes is constant and is given by the number of coils in series $\times$ by the mean E.M.F. in each coil. From §32 we see that this is

$$4\Phi n N \times 10^{-8} \text{ volts,}$$

for a two-pole machine, N being the number of turns in series from brush to brush.

In the case of a multipolar machine, the interval between one north pole and the next corresponds to a whole revolution in the case just considered. Hence, instead of n, we must write np, where p is the number of *pairs* of poles, so that the E.M.F. $=4\Phi Nnp10^{-8}$ ....(17)

§ 34. **Multiplex Windings.** The formula (17) will still be true for multiplex windings, since N has been specified as the number of turns in series, in a single winding, from brush to brush. From this formula we can derive others which are sometimes more convenient.

Let z = the total number of armature conductors.
$2p$ = the number of poles.
m = the number of independent windings.

Then, for lap winding, $N=\dfrac{z}{4mp}$,

and for wave winding, $N=\dfrac{z}{4m}$,

and we have:

$$\text{E.M.F.}=\begin{cases}\dfrac{n\Phi z}{m}10^{-8} \text{ for lap wound armatures,}\\[2ex] \dfrac{pn\Phi z}{m}10^{-8} \text{ ,, wave ,, ,,}\end{cases}$$

where Φ is the total flux per pole.

§ 35. **Example 1.** The armature of a six pole 200 *k*w. generator has a two-circuit winding and rotates at 400 R.P.M. The total number of armature conductors is 480, and the flux issuing from each pole is 5 million lines. Find the E.M.F. generated.

The armature is wave wound, and

$$m=1.$$
$$p=3.$$
$$n=\frac{400}{60} \text{ and } z=480.$$

Hence the E.M.F. is

$$3\times\frac{400}{60}\times 5\times 10^{6}\times 480\times 10^{-8}$$
$$=480v.$$

Example 2. The armature of a four-pole dynamo has 396 conductors in the armature slots. There are only two brushes 90° apart, and there are only two paths through the armature from one brush to another. The commutator has 99 segments. Calculate the flux per pole to give an E.M.F. of 250 volts at 1100 R.P.M. (Mech. Sc. Trip. 1911.)

Using the formula of §34,

we have $E=250,\ p=2,\ z=396,\ n=\dfrac{1100}{60},\ m=1$

$$\Phi=\frac{mE.10^8}{pzn}=\frac{250.10^8.60}{2.396.1100}$$
$$=1,720,000,$$

which is the total flux per pole.

Example 3. A six-pole dynamo has a wave wound armature. The axial length of the armature is 40 cm., its diameter is 30 cm. The poles cover $\frac{2}{3}$ of the periphery; the air gap induction is 6000, and the speed 240 R.P.M. Find the number of conductors to give an E.M.F. of 150 volts. (Inter-coll. Exam., Cambridge, 1911.)

We have:

$$m=1$$
$$E=p\Phi\ n10^{-8},$$

and:

$$E=150\text{v}.$$
$$p=3.$$
$$n=4.$$

Now Φ=flux under one pole,

$$=(6000\times\tfrac{2}{3}\pi\times30\times40)\div6.$$

Hence

$$z=\frac{mE.10^8}{p\Phi n}$$
$$=\frac{150\times10^8}{3\times6000\times\frac{2}{3}\pi\times30\times\frac{40}{6}\times4}$$
$$=496.$$

THE MAGNETIC CIRCUIT OF A DYNAMO.

§ 36. **Bipolar Machines.** The commonest types of field magnets are shown in Fig. 19. The calculations for the Lahmeyer and Manchester types will proceed on the same lines

as in the case of multipolar machines and so will not be dealt with separately. The dotted lines show the general directions of the lines of force.

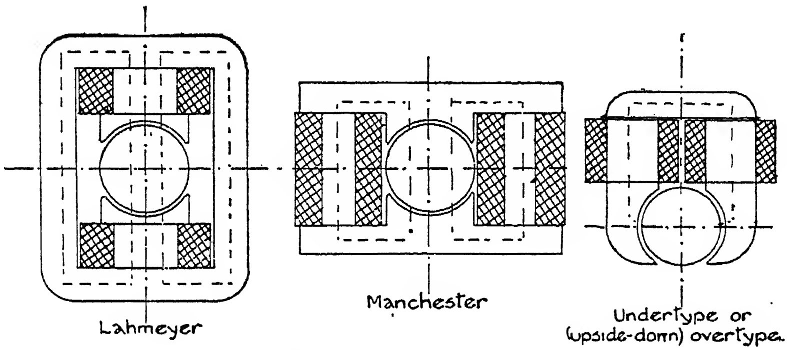


FIG. 19

The problem is to find the number of ampère-turns (NI) required on the magnets to produce a given flux in the air-gap, i.e. a given flux under one pole face. We first find the reluctance of each part of the circuit, which is $\frac{l}{\mu S}$, where l is the mean length of the path, and S the mean cross-section of the path, perpendicular to the lines of force, and multiply this by the flux through that part; this gives the drop of magnetic potential, and by summing up all such drops we obtain the total magneto-motive force required, which we call $\mathscr{F}$. Then we have

$$NI = \frac{10\mathscr{F}}{4\pi} \quad \ldots\ldots\ldots\ldots \quad (18)$$

In calculating the reluctance of the path through the armature, we must allow for the fact that some of the lines coming out of a pole do not go through the armature, but leak outside. This is called *Leakage*, and, if Φ_A is the flux through the armature, and Φ_P the flux in the pole and yokes, we have

$$\Phi_P = \lambda \Phi_A \ldots\ldots\ldots\ldots\ldots (40)$$

where λ is a constant called the *coefficient of dispersion*, or *leakage coefficient*, and varies from about 1·08 to 1·4.

§ 37. **Multipolar Machines.** Fig. 20 shows the field-magnets and armature of a multipolar machine, the armature being hollow; this is generally the case in large multipolar machines, since the flux has not to cross the air inside the armature; additional reasons for making the armature hollow are that it facilitates the cooling and saves weight. The paths

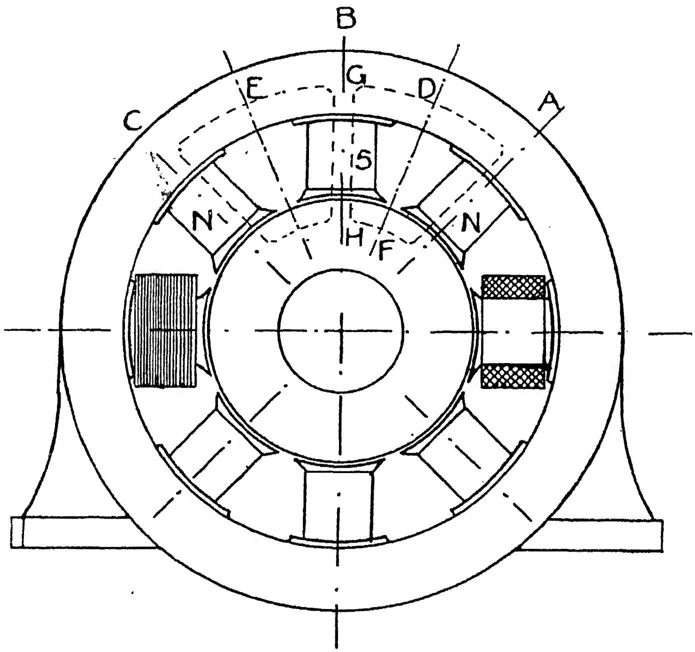


FIG. 20.

of the flux are shown by the dotted lines. Consider any one of the magnetic circuits; its path is from the pole A, through the yoke D, and the pole B, across the air-gap, through the part F of the armature, and again across the air-gap. In parallel with this path is another through the pole C, hence there is twice as much flux in each pole piece as there is through each yoke and each portion of the armature.

To calculate the number of ampère-turns required we see that we must have such a number on pole B that will maintain the difference of magnetic potential necessary to carry the flux through the path from D, through pole B to F, for if the coil will maintain this P.D. on one side of the pole, it will

do so on the other. Hence the number of ampère-turns on each pole must be calculated for the path DGHF.

Leakage occurs from pole-tip to pole-tip; and from pole to pole it is allowed for as in the case of a bipolar machine. Another correction has to be applied for "fringing," i.e. a spreading of the lines from the edge of the pole-shoes, which amounts to an increase in the effective area of the air-gap. This increase may be taken at about 10 per cent. approximately.* Fringing also increases the lengths of the paths of some of the lines crossing the air-gap.

§ 38. **Example 1.** A multipolar dynamo has the following dimensions:

Section of pole ....900 cm^2.
Length of pole ....20 cm.
Section of yoke .... 600 cm^2.
Mean length of path from pole to pole in yoke = 75 cm.
Ditto in armature = 36 cm. Section of paths in armature = 400 cm^2.
Area of pole face ....1200 cm^2.
Width of air-gap ....0·6 cm.
Flux per pole ....10^7 lines.
Find the number of ampère-turns per pole, taking $\mu = 2000$, and neglecting leakage.

1. *The Yoke.* (We only consider half the length of the path from pole to pole.)

$$\text{Reluctance} = \frac{37 \cdot 5}{2000 \times 600}$$

Only half the pole flux goes through the yoke,

$$\therefore \quad \text{drop of magnetic potential} = \frac{10^7}{2} \times \frac{37 \cdot 5}{2000 \times 600} = 156.$$

2. *The Pole.*

$$\text{Reluctance} = \frac{20}{2000 \times 900}.$$

$$\text{Drop of magnetic potential} = \frac{10^7 \times 20}{2000 \times 900} = 111.$$

* See a paper by Mr. F. W. Carter in the *Journal of the I.E.E.*, Vol. 29, for a complete treatment of this subject.

3. *The Air-Gap.*

$$\text{Reluctance} = \frac{0{\cdot}6}{1200}$$

$$\text{Drop of magnetic potential} = \frac{10^7 \times 0{\cdot}6}{1200} = 5000.$$

4. *The Armature Iron.*

$$\text{Reluctance} = \frac{18}{2000 \times 400}.$$

$$\text{Drop of magnetic potential} = \frac{10^7 \times 18}{2 \times 2000 \times 400} = 112.$$

Hence total M.M.F. required $= 156 + 111 + 5000 + 112,$
$= 5379.$

$$\therefore \quad \text{ampère-turns required} = \frac{10}{4\pi} \times 5379 = 4290 \text{ per pole.}$$

Example 2. Fig. 21 shows the dimensions (cm.) of part of a six-pole, shunt wound, 200 K.W., dynamo. The pole shoe embraces an angle of 40°. The coefficient of dispersion is 1·17. The breadth of the slots is equal to the breadth of the teeth at the top, and there are 220 slots. The armature is lap-wound, and each armature slot carries two bars. The magnetic quality of the iron used is shown in Fig. 22. Calculate the ampère-turns required if the E.M.F. is to be 220 V. at 240 R.P.M. (Mech-Sc. Trip., B. Cambridge, 1909.)

The flux per pole in the air-gap is given by

$$220 = \Phi \times 220 \times 2 \times \frac{240}{60} \times 10^{-8},$$

which gives $\Phi = 12{\cdot}5 \times 10^6.$

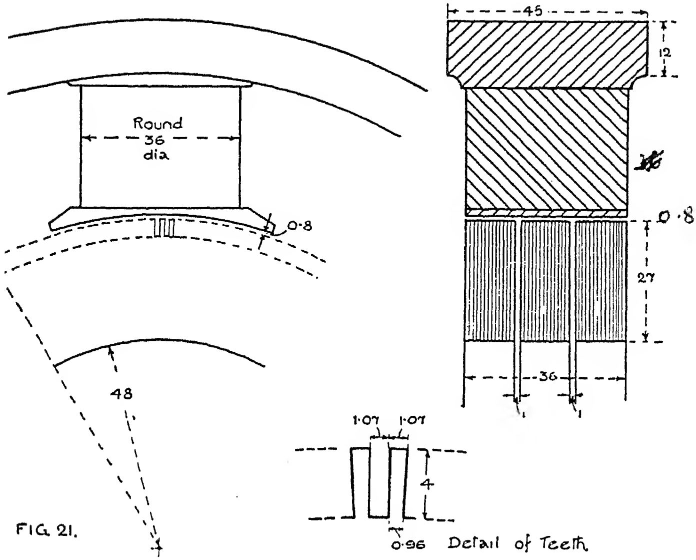
The flux in the pole will be $1{\cdot}17 \times 12{\cdot}5 \times 10^6 = 14{\cdot}62 \times 10^6$ ✓
The sectional area of the pole $= \pi . 18^2 = 1020$ cm^2.

$$\therefore \quad \text{the } \textit{density in the pole} = \frac{14{\cdot}62 \times 10^6}{1020} = 14{,}330.$$

The flux in the yoke $= \frac{1}{2}$ the flux in the poles, $= 7{\cdot}31 \times 10^6$.
The area of the cross section of the yoke $= 540$ cm^2.

$$\therefore \quad \text{the } \textit{density in the yoke} = \frac{7{\cdot}31 \times 10^6}{540} = 13{,}540.$$

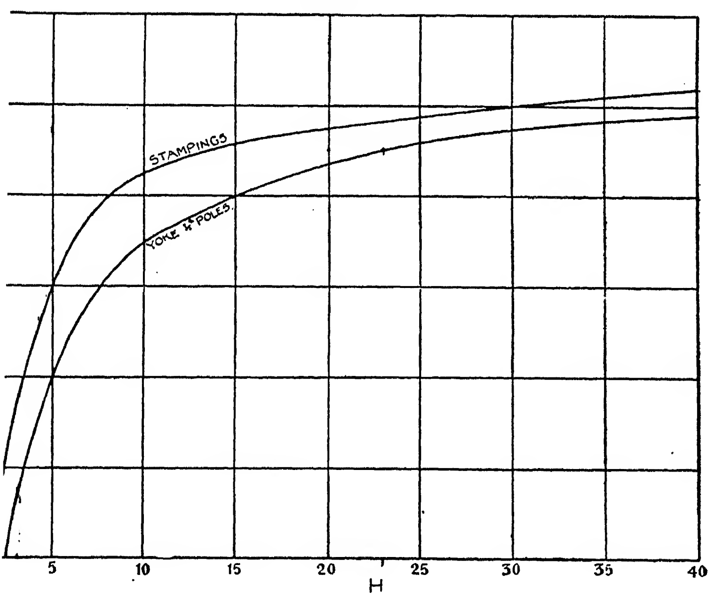
Air-Gap Density. The area of the air-gap may be, in the first place, estimated as the mean between the area of the pole-face and the area of the tops of all the teeth under the pole.



45
12
Round
36
dia
0·8
0·8
27
36
48
1·07
1·07
4
0·96
Detail of Teeth

FIG. 21.

FIG 22



STAMPINGS
YOKE & POLES.
5
10
15
20
25
30
35
40
H

The area of the pole-face $=\dfrac{40\pi}{180}\times 75\cdot 8\times 36=1900$ cm²

The area of the top of one tooth $=1\cdot 07\times 3\overset{4}{\cancel{8}}=35\cdot 4$ cm².

$\therefore$ the area of the tops of all the teeth under one pole

$$=\frac{40\times 220}{360}\times 35\cdot 4=865 \text{ cm}^2.$$

The mean of 865 and 1900 is 1382 cm².

To this we add 10 per cent. for fringing, i.e. 138.

$\therefore$ the effective area of the air-gap $=1520$ cm².

$\therefore$ the air-gap density $=\dfrac{12\cdot 5\times 10^6}{1520}=8230.$

Tooth Density. Take the area of the teeth as the mean between the area at the top and the area at the bottom. The mean width of a tooth $=1\cdot 015$ cm.

$\therefore$ the mean tooth area, for all the teeth under one pole, is $\dfrac{40\times 220}{360}\times 1\cdot 015\times 34=843$ cm².

$\therefore$ the tooth density $=\dfrac{12\cdot 5\times 10^6}{843}=14{,}840.$

Armature Density. The area of the section of the armature below the teeth $=23\times 34=782$ cm². The flux $=\frac{1}{2}\times 12\cdot 5\times 10^6$.

$\therefore$ the armature density $=\dfrac{6\cdot 25\times 10^6}{782}=7990.$

This assumes that the flux density in the armature is constant, which is not true, but the ampère-turns required for the armature reluctance are a small part of the whole, and so we can neglect small corrections in estimating them.

The lengths of the Magnetic Paths (see Fig. 23).

$$AB=\frac{2\pi\times 114}{12}=72 \text{ cm}.$$

$$BC=42 \text{ cm}.$$

$$CD=0\cdot 8 \text{ cm}.$$

Depth of teeth $=4$ cm.

$$GEF=\frac{2\pi\times 61\cdot 5}{12}-9+\frac{\pi}{2}\times 9,$$

$$=32\cdot 3-9+14\cdot 1,$$

$$=37\cdot 4 \text{ cm}.$$

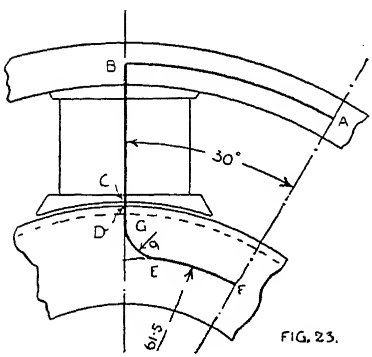


FIG. 23.

Calculation of the ampère-turns. This can now be done in a table, with the help of Fig. 22.

Part	Flux Φ	Section S	Flux Density	$\mathcal{H}$ From Fig. 22	$\mu = \mathcal{B}/\mathcal{H}$	Length l	$\Phi \frac{l}{\mu S}$
Yoke	$7{\cdot}31 \times 10^6$	540	13,540	12·3	1,100	72	885
Pole	$14{\cdot}62 \times 10^6$	1,020	14,330	16·8	853	42	705
Air-gap	$12{\cdot}5 \times 10^6$	1,520	8,230	8,230	1	0·8	6,584
Teeth	$12{\cdot}5 \times 10^6$	843	14,840	12	1,237	4	48
Armature	$6{\cdot}25 \times 10^6$	782	7,990	2·6	3,070	37·4	97
				∴ The total M.M.F. required			8,319

$$\therefore \text{ the ampère-turns required} = \frac{10}{4\pi} \times 8319$$
$$= 6620 \text{ per pole.}$$

LEAKAGE.

§ 39. We shall now consider the question of leakage in more detail. The amount of leakage flux can be only roughly estimated, but the standard cases here given are sometimes useful. They are calculated on the assumption that the reluctance of the iron part of the path may be neglected.

§ 40*. **Equal and Parallel surfaces in the same plane, and separated by a short gap.**

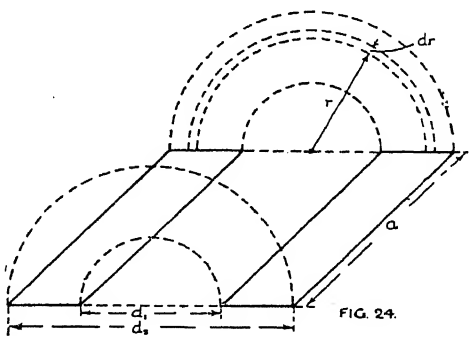


FIG. 24.

Let $\mathcal{P}$ = the permeance of the path of leakage flux, i.e. the reciprocal of the reluctance, then it is easy to show that we can add the permeances of magnetic paths in parallel, in the same way that we add the conductance of electrical paths in parallel.

* This and §41 are due to Prof. Forbes.

Assume the lines of force to be semi-circles, and consider an elementary cylindrical path, of radius r, and thickness dr.

The permeance of this is $(\mu = 1)\dfrac{adr}{\pi r}$ (see Fig. 24).

Hence, for the whole leakage path,

$$\mathscr{P} = \int_{\frac{d_1}{2}}^{\frac{d_2}{2}} \frac{adr}{\pi r} = \frac{a}{\pi} \log_e \frac{d_2}{d_1} \quad \ldots\ldots\ldots\ldots \quad (19).$$

and the leakage flux $= \dfrac{4\pi}{10} \times$ amp.-turns $\times \mathscr{P}$

§ 41. **Equal and Parallel Faces in the same Plane, but separated by a large gap.** (Fig. 25.) In this case we assume

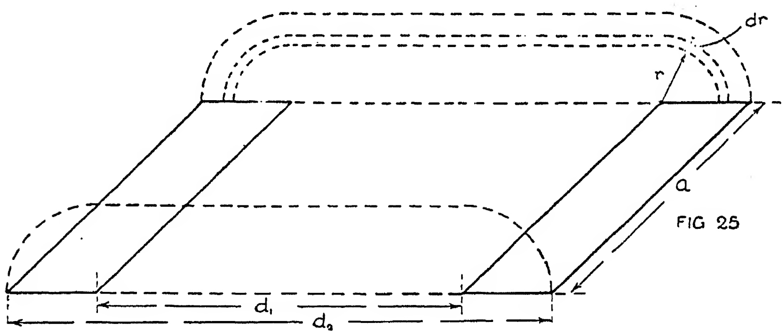


FIG 25

the lines of force to be straight lines, ending in quadrants of circles, struck with the inner edges of the faces as centres.

We have

$$d\mathscr{P} = \frac{adr}{\pi r + d_1}$$

and

$$\mathscr{P} = \int_0^{\frac{d_2 - d_1}{2}} \frac{adr}{\pi r + d_1}$$

$$= \frac{a}{\pi} \Big[\log_e (\pi r + d_1) \Big]_0^{\frac{d_2 - d_1}{2}}$$

$$= \frac{a}{\pi} \log_e \frac{\pi(d_2 - d_1) + 2d_1}{2d_1} \quad \ldots\ldots\ldots \quad (20).$$

If the surfaces are not parallel, either in this or the previous case, but inclined at angle θ (radians), we must substitute θ for π in the above results.

§ 42. **Leakage between Field Magnets of Multipolar Machines.*** (Fig. 26.) The flux is assumed to follow straight line paths such as AB, CD, as this gives results which are more in accordance with those derived from experiment than when the path is taken as circular.

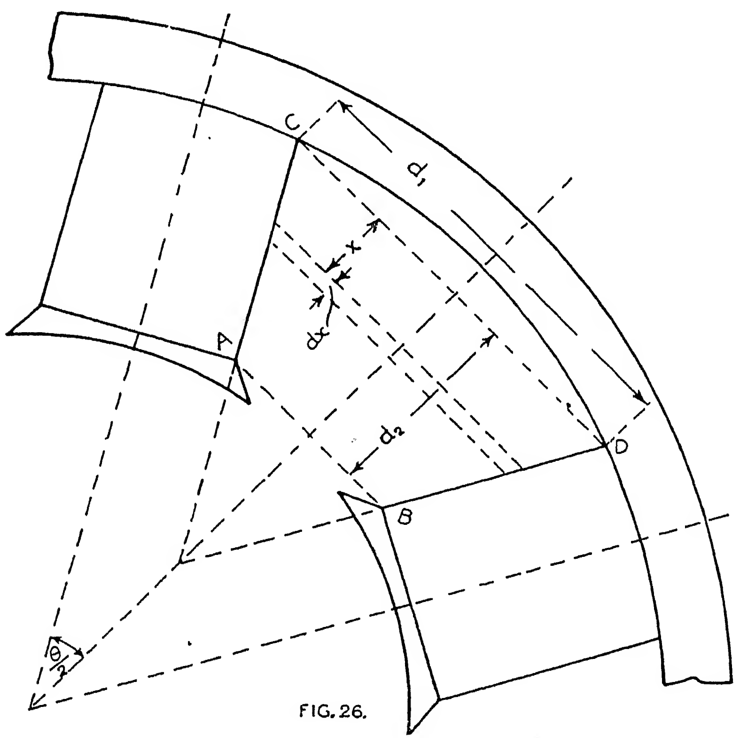


FIG. 26.

Let a = depth of magnets, perpendicular to the plane of the paper.

Let the number of wires on AC and BD together be z per unit length, each carrying 1 amp. Then the flux across any section adx is

* W. Cramp.

$$\frac{4\pi}{10}.z.\frac{a\,dx}{d_1-2x\tan\frac{\theta}{2}}$$

$\therefore$ the total flux

$$=\frac{4\pi}{10}za\int_0^{d_2}\frac{x\,dx}{d_1-2x\tan\frac{\theta}{2}}$$

$$=\frac{4\pi}{10}za\left[\frac{d_1}{4\tan^2\frac{\theta}{2}}\log_e\frac{d_1}{d_1-2d_2\tan\frac{\theta}{2}}-\frac{d_2}{2\tan\frac{\theta}{2}}\right]\ldots\ldots\ldots\ldots(20).$$

ARMATURE REACTION.*

§ 43. We have considered the armature and the field magnets separately, and now we must deal with their effect on one another.

Since the armature has, on its periphery, a number of conductors carrying current, it will itself produce a magnetic field, which must affect the field due to the field magnets. The effect which the armature field has on the main field is called armature reaction.

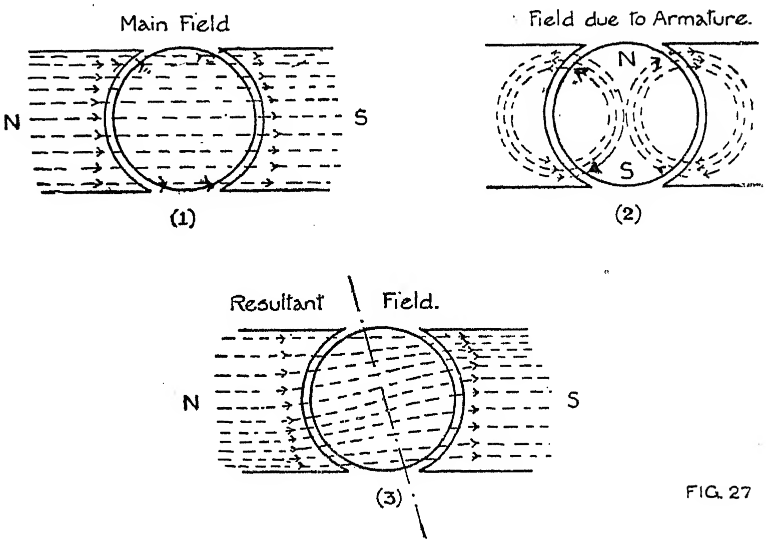


FIG. 27

* See also *Mémoirs sur L'Electricité et L'Optique par Potier.* Blondel.

§ 44. *Armature Reaction.* The two fields are shown separately, in a diagrammatic manner, in (1) and (2), Fig. 27, from which it will be seen that at ~~A~~ B and ~~C~~ D the two fields oppose one another, and so the resultant field is weaker at these two points; similarly it is stronger at ~~B~~ A and ~~D~~ C. The resultant field is shown in (iii), Fig. 27. The neutral line, i.e. the line through the axis normal to the resultant field, is displaced in a forward direction, and so the brushes are moved on by the same amount. In the case of a motor the displacement is opposite to the direction of rotation, since, with the same polarities, the armature will be turning the other way.*

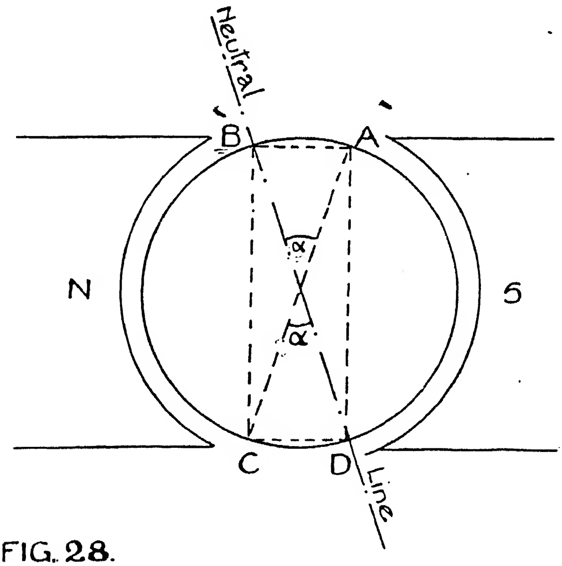


FIG. 28.

§ 45. Now let us examine the consequences of this. Let BD be the new position of the neutral line and brushes; draw AC (Fig. 28) symmetrical with this line, and let $2a$ be the angle between them. The turns included within the angle $2a$ produce a field which is directly opposed to the main field, and so are called *Demagnetising turns*. The rest of the turns, i.e. those included in the two angles $\pi - 2a$ produce a field at right angles to the main field, and are called *cross turns*. The demagnetising turns weaken the field, and the others distort

*See § 78.

it, and we see that the more the brushes are moved forward the more is the main field weakened. Moreover, the greater the current delivered by the dynamo, the greater will the demagnetizing ampère-turns become, and so the weakening increases with the load.

Let N be the total number of armature turns, i.e. half the number of conductors, and let $a°$ = the amount the brushes are moved forward, i.e. the "lead."

Then the number of demagnetizing turns $= \frac{2a}{180}N$. In a two-pole machine, the current in each conductor is half the total current.

Let I ampères be the total current. Then the number of demagnetising ampère-turns is

$$\frac{I}{2}\frac{2a}{180}N = \frac{aIN}{180} \quad \dots\dots\dots\dots\dots\dots (22).$$

This number of extra ampère-turns must be provided on the field, to prevent a reduction of air gap flux by the reaction.

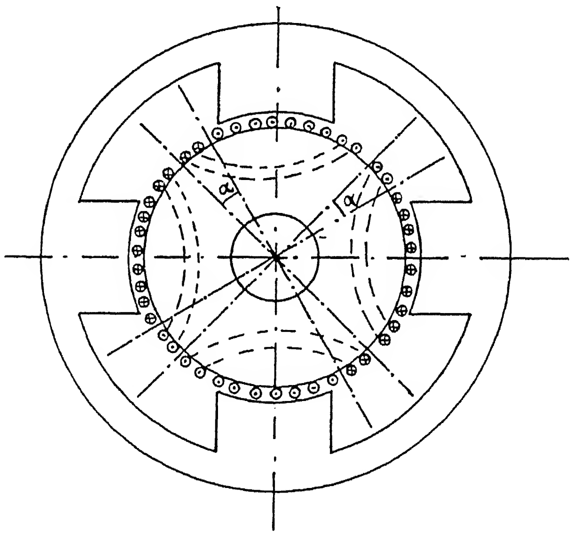


FIG. 29

§ 46. *Armature Reaction in Multipolar Machines.* If the brushes are moved from the mid position through an angle

$a°$, the current has the directions shown, and the demagnetizing turns are those within an angle $2a$, as shown by the dotted lines, and we have: demagnetising ampère-turns *per pole* = current in each conductor $\times \dfrac{aN}{180}$

§ 47. **Example 1.** A two-pole, 200 volt, 10 *k*w. dynamo has 200 conductors on the armature, and the lead is 18°. Find the demagnetizing ampère-turns on the armature at full load. (Inter. Coll. Exam., Cambridge, 1904.)

The number of demagnetizing turns

$$= \frac{2a}{180}N$$

$$= \frac{2 \times 18}{180} \times 100 = 20.$$

The current in each turn $= \frac{1}{2} \cdot \dfrac{10{,}000}{200} = 25$A.,

$\therefore$ the demagnetizing amp.-turns are 20×25, i.e. 500.

Example 2. The following are particulars of a 200 *k*w. generator: Six poles, 500 volts, 474 armature conductors, two-circuit winding; brush lead 4°. Calculate the number of demagnetizing ampère-turns at full load.

Total number of turns on armature = 237.

„ current $= \dfrac{200{,}000}{500} = 400$A.

Since the winding is two-circuit, the current in each turn will be 200A.

Hence the demagnetizing ampère-turns per pole

$$= 200 \times \frac{4}{180} \times 237,$$

$$= 1050.$$

§ 48. **Interpoles.** We see that, if we could overcome the effects of the cross ampère-turns, we should have no distortion of the main field, consequently there would be no brush displacement, and therefore no demagnetization. The field due to the cross ampère-turns is at right angles to the main field; if, then, we provide another field at right angles to the main field, but in a direction opposite to the field due to the

cross ampère-turns, the effects of armature reaction would be neutralized. The most common way of doing this is by

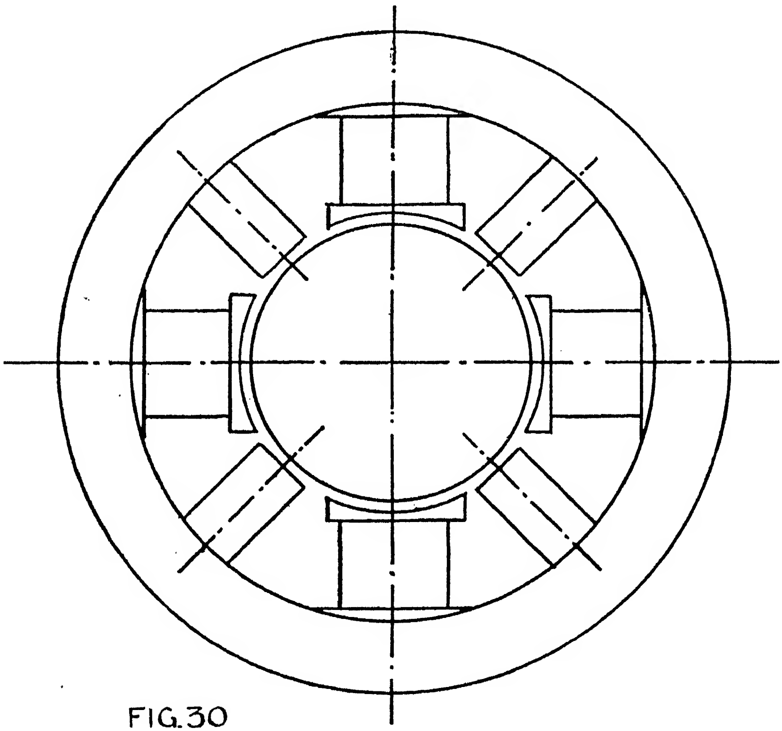


FIG. 30

providing small auxiliary poles, called *interpoles* or "*Reversing poles,*" between the main poles, as shown in Fig. 30, the smaller poles being the interpoles.

§ 49. **Flux Distribution in the Air-Gap.** The effect of armature reaction on the air-gap flux can be seen by measuring the flux density at different points in the gap, which can be done as follows:

An extra, temporary, brush-holder is required, and this carries two brushes separated by a distance equal to that between two consecutive commutator segments, and connected to a voltmeter. All the armature coils pass under these brushes in turn, so that the voltmeter shows the voltage induced in the coils passing through that particular part of the field which the temporary brushes happen to occupy, and this voltage is proportional to the flux density at that point. Hence

if we set these two brushes at successive points round the commutator, we can determine the distribution of flux in the air-gap.

When there is no current in the armature the flux distribution, or field-form curve, is usually somewhat as shown in Fig. 31, being constant over the space under each pole, and completely changing in direction in the interval between the poles.

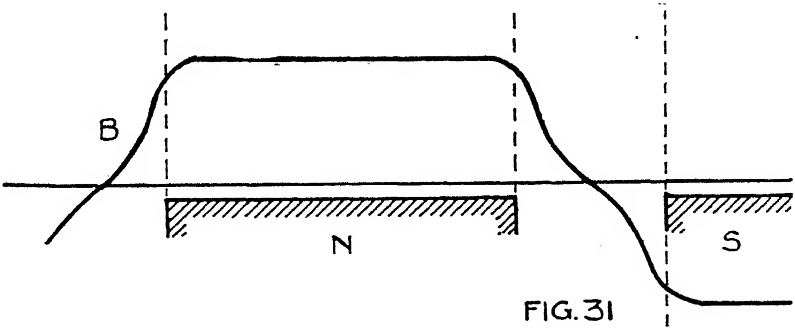


FIG. 31

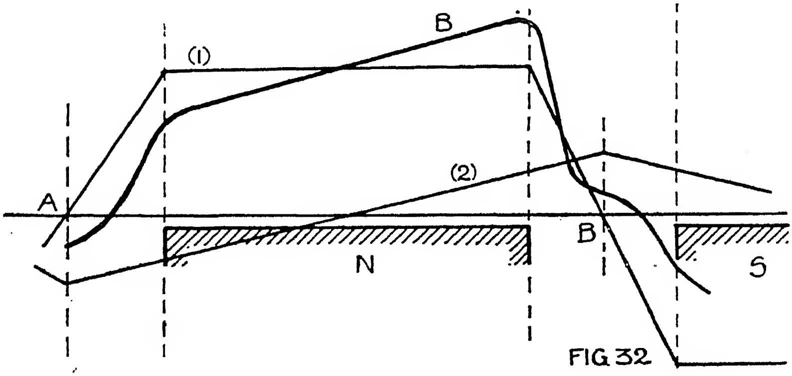


FIG. 32

When loaded, i.e. when there is a current in the armature, the flux distribution is changed, owing to the magnetizing force of the armature current. In Fig. 32 the curve (1) represents the magnetizing force of the poles, and (2) that of the armature, the brushes being at the points A and B, midway between the poles. The resultant magnetizing force is obtained by adding these two, and from this the curve of flux density ($\mathscr{B}$) is obtained.

When the brushes are moved forward the state of affairs is rather different. Suppose the brushes are moved on to the tip of the next pole, as in Fig. 33, where the brushes are supposed to be at A, B. In Fig. 32, the M.M.F. of the armature helps the main M.M.F. in one half of the pole as much as it

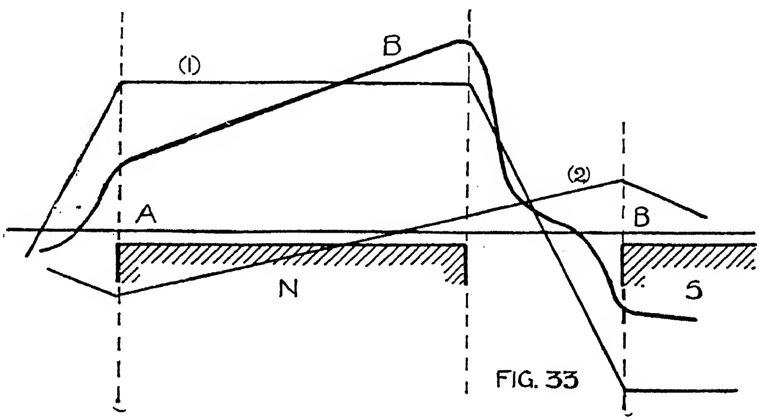


FIG. 33

opposes it in the other, so that there is, on the whole, no demagnetization; but in Fig. 33, when the brushes are not in the central position, the armature M.M.F. is in opposition over a range greater than that in which it is additive, so that the nett result is a weakening of the resultant field.

COMMUTATION.

§ 50. When a pair of commutator segments pass under a brush, the current in the coil connected to those segments has to be completely changed in direction, and this must be effected during the short time the segment is under the brush. If it is not reversed by the time the segment leaves the brush, a spark will result, since the reversal cannot be instantaneous, owing to the effect of self-induction, which tends to cause a spark whenever a circuit is suddenly broken. This process of reversing the current is called commutation, and it is very desirable that it should not be attended by sparking, which is detrimental to the machine.

One way to reduce sparking is to use carbon brushes which have a very high resistance.

Suppose the commutator moving in the direction shown by the arrow in Fig. 34, where the coils underneath represent the armature coils, and the current in 2 is undergoing reversal. Consider position I, where *a* has just come under the brush ;

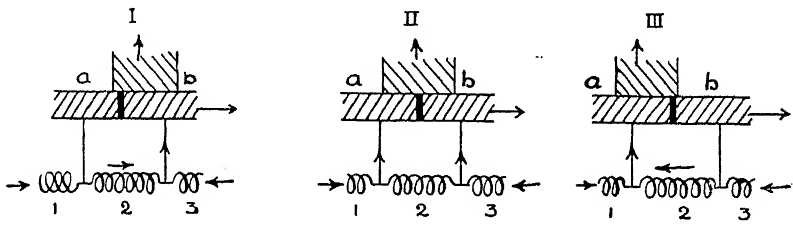


FIG. 34

the contact resistance is very great, so that nearly all the current goes through *b*, 2 carrying most of the current from 1. When the position is symmetrical (II) *a* and *b* carry an equal amount of current, and 2 has none. In position III, the contact resistance between *b* and the brush is great, so most of the current from 3 will go through 2. Hence the current in 2 has been completely reversed while it has passed under the brush.

§ 51. **Commutating Poles.** Another method of obtaining sparkless commutation, known as E.M.F. commutation, is to reverse the current in the coil by an opposing E.M.F., which is induced by the use of interpoles. To calculate the required E.M.F. we proceed as follows :

Let L = coefficient of self-induction of the short circuited coil.
r = resistance of ditto.
t_c = the time it is short circuited.
i = the current in it.
i_0 = the maximum value of this current.
E_c = the E.M.F. required for commutation, and produced by the flux from the interpoles.

Then we have $ir + L\dfrac{di}{dt} + E_c = 0$

whence $$\frac{rt}{L} = \log_e A - \log_e \frac{ir + E_c}{r}$$

where A is a constant to be determined.

At the beginning of commutation, $t=0$, $i=i_o$.
,, ,, end ,, ,, $t=t_c$, $i=-i_o$.

Hence we have $A=\dfrac{i_0 r+E_c}{r}$

and $$\frac{rt_c}{L}=\log_e\frac{E_c+i_0 r}{E_c-i_0 r}$$

$\therefore$ $$E_c=i_o r\,\frac{1+e^{\frac{-rt_c}{L}}}{1-e^{\frac{-rt_c}{L}}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3),$$

this is called the *reactance voltage*.

We shall also have $E_c=2N\Phi 10^{-8}/t_c$ ....(24)
where N is the number of turns in each coil, and Φ is the flux necessary in the interpole. Hence we can find Φ, and so the number of ampère-turns necessary on the interpoles.

In the derivation of the above formula it has been assumed that r is constant, i.e. no allowance has been made for the changing area of brush contact. This is allowable since, with properly designed interpoles, it should not be necessary to use high resistance brushes.

t_c, the time of commutation, is calculated from

$$t_c=\frac{\text{width of brush, measured in commutator segments}}{\text{speed of commutator in segments /sec.}}$$

and $L=$flux per amp. in 1 turn $\times N^2\times 10^{-8}$.

§ 52. **Other methods of Calculating the Commutation E.M.F.** Hobart suggests that the current curve, during commutation, be taken as sinusoidal, in which case we have,* neglecting the resistance.

$$E_c=i_0\,.\,2\pi f_c L \qquad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

where $f_c=$the frequency of commutation,

$$=\frac{\text{peripheral speed of commutator}}{2\times\text{width of brush}}.$$

If the current curve be taken as a straight line, we have*

$$E_c=\frac{2}{\pi}\times\text{previous value.}$$
$$=4i_0 f_c L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

* See *Continuous Current Machine Design.* W. Cramp.

In calculating the total number of ampère-turns for the interpoles we must add to those required to produce the commutation flux a number equal to the number of armature cross ampère-turns, as these have to be neutralized before the commutation flux can be produced. The leakage factor for interpoles is usually large—often about 1·8.

For purposes of calculation, a curve showing the connection

between $\frac{rt}{L}$ and $e^{-\frac{rt}{L}}$ is given in Fig. 35.

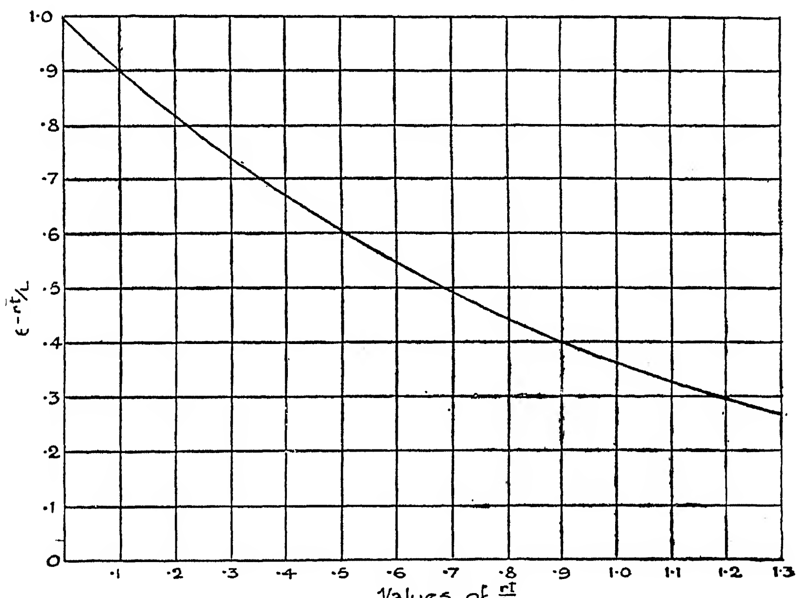


FIG. 35.

The value of E_c found above is a reactance voltage in the coil undergoing commutation. The difficulty in estimating its value lies in the calculation of L, which can really only be arrived at from the results of experiment.*

§ 53. **Example.** Referring to the machine of p. 31, the number of commutator segments is 220, and there is one turn per armature coil. The brushes cover two commutator segments. The armature is lap-wound, and the flux produced

* See *Jour. Inst. E.E.*, vol. 31, p. 180, and Cramp's *Continuous Current Machine Design*, chap. ix.

per cm. of imbedded conductor per ampère is 9 lines. The resistance of a single coil is 0·0002 ohm. Calculate the Reactance Voltage and the commutating flux necessary, by the method of § 51. (Mech. Sc. Trip. 1909, B.)

$$\text{The total current at full load} = \frac{200{,}000}{220} = 909 \text{ A.}$$

$$\therefore \quad \text{the current in each circuit} = \frac{909}{6} = 151 \cdot 5 \text{ A.}$$

$L =$ the coefficient of self-induction of each coil

$$= 34 \times 9 \times 1^2 \times 10^{-8} = 306 \,.\, 10^{-8} \text{ Henries.}$$

$$r = 2 \times 10^{-4} \text{ ohms.}$$

$$t_c = \frac{2}{220 \times 4} = 2 \cdot 28 \times 10^{-3} \text{ secs.}$$

$$\frac{rt_c}{L} = \frac{2 \times 10^{-4} \times 2 \cdot 28 \times 10^{-3}}{306 \times 10^{-8}} = 0 \cdot 149$$

$$\epsilon^{-\frac{rt}{L}} = 0 \cdot 86$$

$$E_c = 151 \cdot 5 \times \cdot 0002 \times \frac{1 \cdot 86}{0 \cdot 14}$$

$$= 0 \cdot 403 \text{ volts.}$$

$$\text{And the interpole flux} = \frac{E_c t_c 10^8}{2N}$$

$$= \frac{0 \cdot 403 \times 2 \cdot 28 \times 10^{-3} \times 10^8}{2} = 46{,}000 \text{ lines.}$$

THE PERFORMANCE OF DYNAMOS.

§ 54. There are three main classes of dynamos:

(i) **Shunt Dynamos**, in which the field windings are connected across the armature terminals.

(ii) **Series Dynamos**, in which the field windings are in series with the armature and the load.

(iii) **Compound Dynamos**, which have some of the field turns connected across the armature, and some in series with it.

In some cases, and frequently for test purposes, the machines are **separately excited** from an external source, i.e. the field-winding is connected to some other source of power.

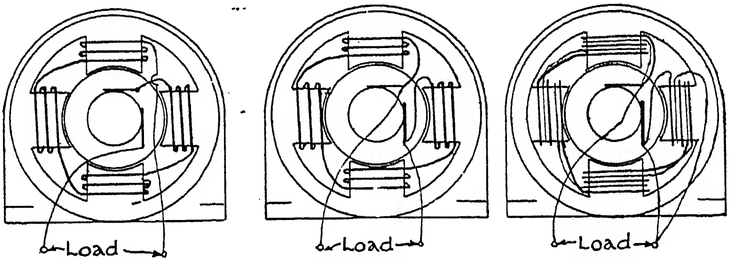


FIG. 36

Although the series-wound machine is of great importance as a motor, as a generator it is seldom met, since it does not satisfy the condition of constant voltage required for most commercial purposes; very little, therefore, will be said about it until we deal with motors.

CHARACTERISTIC CURVES.

§ 55. In dealing with the performance of generators we have, in general, six quantities to consider:

(i) The current in the armature (I_a).
(ii) The current in the shunt coils (I_s).
(iii) The current in the series field coils (I_f).
(iv) The current delivered to the load (external current), I.
(v) The E.M.F. generated (E).
(vi) The P.D. across the terminals (V);

and also the various resistances (r_a, r_s, r_f, R respectively).

Except in compound machines, only one of (ii) and (iii) exists.

§ 56. We shall investigate the behaviour of generators by means of curves connecting certain of the above quantities.

These curves are called *characteristics*, the principal of which are :—

(i) *The Saturation Curve*, *no load characteristic*, *open circuit characteristic*, or *magnetization curve*, showing the relation between E and I_s.*

(ii) *The Total Characteristic* ; a curve between E and I_a.

(iii) *The External Characteristic*, connecting I and V.

(iv) *The Short Circuit Characteristic*, which shows the relation between I_s and I_a when the armature is short-circuited through an ammeter, and the field is separately excited.

In order that the characteristic curves for any particular machine may be definite, the speed of the armature must be defined. Unless otherwise stated, in what follows, it is assumed that the speed is the constant speed at which the machine is designed to run under normal working conditions.

§ 57. **Excitation.** It will be seen from the definitions that the curves (i) and (iv) above will be the same whether the machine is ultimately to be connected as shunt, series, or compound wound, or separately excited, since in any case the quantities plotted are obtained with separate excitation (see § 58). Owing to the fact that the field magnets, when they have once been magnetized, will always retain a small amount of *remanent* or residual magnetism unless subjected to a demagnetizing effect, neither the open-circuit or the short-circuit characteristic start precisely at the origin of co-ordinates ; when the field current is zero the remanent magnetism will be the means of inducing a small E.M.F. in the armature. However, very little error is made by assuming that both curves start as straight lines through the origin. The practical importance of remanent magnetism is that it renders possible self-excitation (see § 67).

§ 58. **Experimental Determination of Short and Open Circuit Characteristics.** For obtaining the open circuit characteristic of a machine the connections are shown in Fig. 37. We drive the machine at the proper speed with different exciting currents and read the armature voltage for each value of

* In a series dynamo there is no shunt current, and the saturation curve is obtained by disconnecting the series coil from the armature, and exciting it separately.

the field current. The characteristic is obtained by plotting the armature volts on a base of field current. For any given

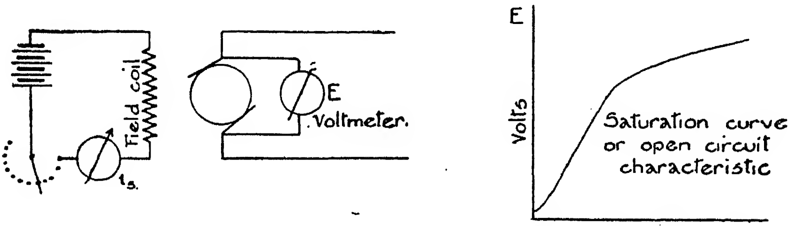


FIG. 37.

value of the field current the flux will be constant, so that the E.M.F. will be proportional to the speed. Hence, if the speed be not quite constant, a correction must be applied to the observed values of E on this basis, before plotting the curve between E.M.F. and excitation.

The no-load curve for a four-pole shunt dynamo is shown in Fig. 40, on which are inscribed the particulars of the machine to which it refers. This diagram also shows the influence of the amount of clearance between the armature and pole pieces.

The connections and procedure for obtaining the short circuit characteristic are essentially the same except that an ammeter replaces the voltmeter and the field currents employed are much smaller.

§ 59. Experimental Determination of External Characteristic.

(i) *Series-Wound Machine.*

We have $I_f = I_a = I$.

If we run the machine at a constant speed, with different resistances in the external circuit, and measure V for different values of I, we obtain the curve OB in Fig. 38. Now, if we draw OA so that, on the proper scales,

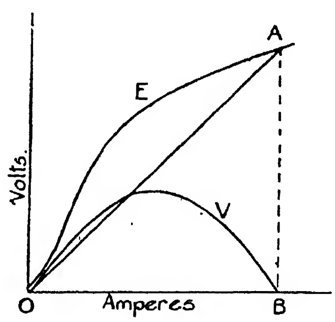


FIG. 38.

$$\tan \alpha = r_a + r_f,$$

the ordinates of OA will show the pressure used in overcoming the resistance of the machine itself. Adding the ordinates of

OA to those of curve V we obtain a curve showing the total E.M.F. (E) generated.

(ii) *Shunt Wound Machine.* The normal connections are as shown in Fig. 39. Using these connections, we keep the speed constant and read the terminal pressure for different values of the current supplied to the load, by varying the latter By this means we obtain a curve such as shown in Fig. 41.

§ 60. It is of course the External Characteristic which is ultimately of importance to the user, but its direct experimental determination is often inconvenient and necessarily expensive in the case of large machines, since the full power has to be supplied and absorbed. Fortunately, however, the open-circuit and short-circuit characteristics, obtained with but small expense for power, enable us to predict the external characteristic with fair accuracy. This we shall now examine in detail in the important case of the simple shunt-wound machine.

CHARACTERISTICS OF SHUNT MACHINES.

§ 61. **The Effect of Armature Resistance.** From Fig. 41 it will be observed that, as the load increases, the P.D. at the terminals falls below the no-load value.

As the armature current increases, the armature drop, $I_a r_a$, i.e. the volts used in sending the current through the armature, increases, so that V, the P.D., is reduced. Moreover, this decrease of V causes a slight decrease in the shunt current, so that the E.M.F. is slightly reduced owing to the smaller flux.

§ 62. **The effect of Armature Reaction.** We have here to consider another cause which reduces the P.D., by decreasing the E.M.F. generated.

It was shown, in § 45, that the effect of the armature current is to produce a field which may be regarded as having a component opposed to the main field, and another component across the main field. It is with the former of these that we are here concerned. When the machine is on open circuit there is no current in the armature, and consequently no opposing armature field. With a current flowing through the armature, the backward field of the armature comes into play, and reduces the effective value of the flux from the field

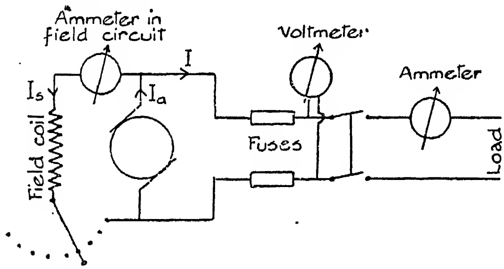


FIG. 39.

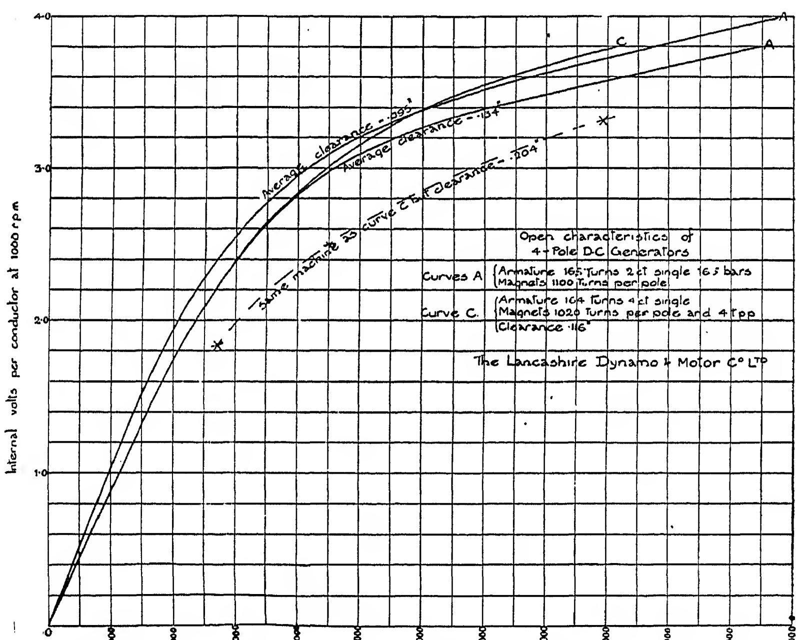


FIG 40

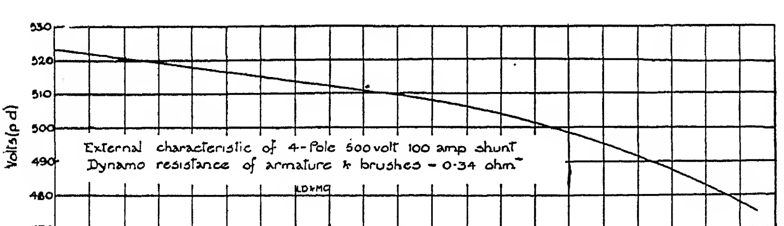


FIG. 41. Load

magnets. Consequently the E.M.F. is reduced, and therefore also the terminal P.D.

In order to estimate the weakening effect on the field, due to armature reaction, we make use of the short circuit characteristic; as indicated above, we obtain this by running the machine at its constant normal speed, with the field separately excited, and the armature short-circuited through an ammeter of very low resistance, and observations are taken of the field current and the armature current. Under these conditions the E.M.F. of the machine is equal to $I_a r_a$, if we neglect the resistance of the ammeter and its connections.

Now, the open circuit characteristic will tell us how much field current is required to produce this pressure ($I_a r_a$) on open circuit, when there is *no* armature reaction. The short-circuit test shows what field current is required to produce the same pressure when there *is* armature reaction, and it will be found that the field current in this case exceeds that when the armature circuit was open. Hence, the difference between the two must stand for the amount of field current in the shunt circuit which is required to balance the effect of armature reaction, and must always be allowed for in estimating the

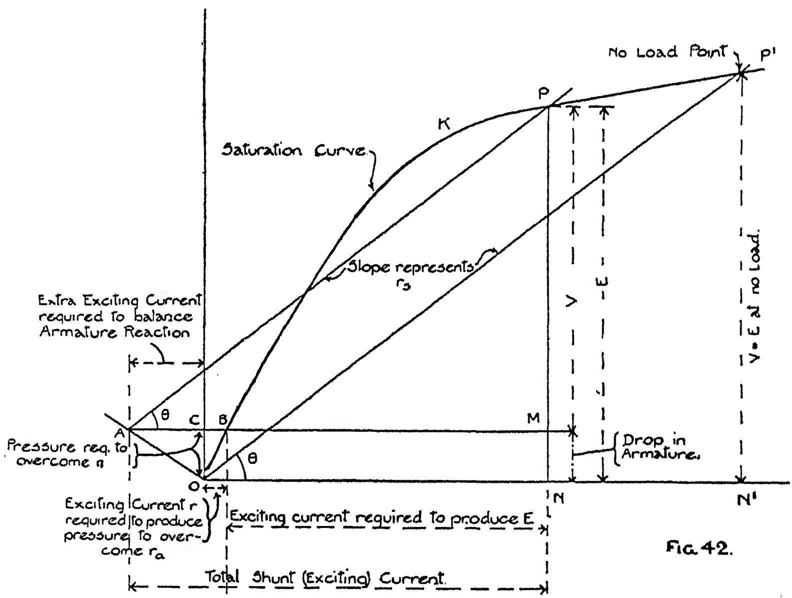


FIG. 42.

field current required to produce any given E.M.F. with a certain current flowing in the armature.

§ 63. This is shown graphically in Fig. 42,* where OK is the open circuit curve.

If we take $OC = I_a r_a$, CB will represent the field current required to produce this pressure. On short-circuit, with the same armature current, it requires a greater field current to generate this pressure OC. Set back this value of the field current from B, as shown by BA in the figure. Then CA is the field current required to neutralize the effect of the armature reaction of I_a.

Now, suppose we require the P.D. to be V, when carrying this current in the armature. The E.M.F. must be $E = V + I_a r_a$, represented by PN. This requires, on open circuit, a field current ON, but we have to overcome armature reaction as well, which requires an amount of current in the field = CA. Hence the total field current required is AC + ON = AM.

Since $OC = I_a r_a$ and $PN = E$, PM will be the terminal P.D. V

And the resistance of the shunt must be given by

$$\frac{V}{I_s} = \frac{\mathrm{PM}}{\mathrm{AD}} = \tan\theta.$$

If this is constant, at no-load the P.D. will be P′M′, where OP′ is parallel to AP.

§ 64. In applying the construction of Fig. 42 to problems a difficulty arises in the matter of scale, for OC will usually be very small compared with PN. This may be overcome in the following manner (see Fig. 43): Let O′ be the point on the saturation curve corresponding to no-load ($V = E$, $I_a = 0$). Take new axes through O′, and draw the first part, OH, of the saturation curve again, as shown by O′H′.

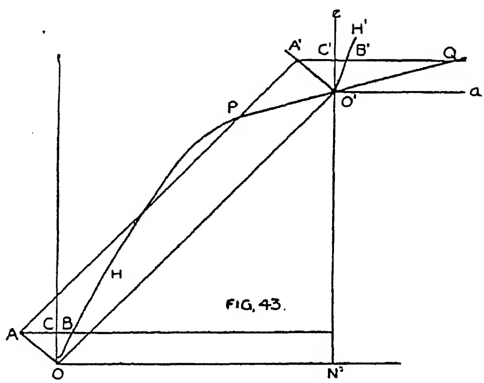


FIG. 43.

* This construction is due to Mr. H. Rottenberg

Draw O′C′, B′C′A′, corresponding to OC and BCA respectively. Then AA′ is parallel to O′O. The point P gives the new value of the E.M.F., and in order to find V we have only to subtract the drop $I_a r_a$.

If V is to be the same at no-load and when a current I_a flows in the armature, produce A′B′ to meet the saturation curve in Q, which determines the new value of the field current.

By this modification of the figure we have only to draw a much smaller figure, using O′a, O′e as axes, so that a more open scale may be employed.

§ 65. **To derive the Total Characteristic and External Characteristic.** From this figure we can readily obtain a series of corresponding values of E and I_a. If we plot these, we have the Total Characteristic, and from this we can deduce the External Characteristic as follows: Subtract the shunt current from the total armature current (I_a), and this gives us the external current; then reduce the corresponding E by the armature drop, and we have V, or we may read V direct from the figure, e.g. PM. in Fig. 42. If we now plot a curve of V and external current (I) we have the External Characteristic. It is important to notice that the locus of A for different values of I_a is practically a straight line, since (i) $OC = I_a r_a$, and (ii) AC represents the amount of shunt current, which, acting in a *fixed* number of shunt turns, balances the demagnetizing ampère turns on the armature, and these are proportional to I_a so long as the lead of the brushes is not altered. Thus OC and CA are proportional to I_a and the locus of A is a straight line. Consequently, given one position of A, we can draw the line for any given value of the brush lead.

The method of devising the Total and External Characteristics in the case of a separately excited generator may easily be developed along similar lines, and is left as an exercise for the student.

§ 66. **Example 1.** A shunt machine has a saturation curve as below: A short circuit current of 100 amps. is given by 0·6 amp. in the shunt, and the armature resistance is 0·05 ohm. If the P.D. is to be 150 volts throughout, find the total shunt resistance at no-load, 25, 50, 75 and 100 amps. in the armature.

Saturation Curve, 4A. gives 150 volts, 6A. gives 155 volts, straight between. The first part is straight and such that 20 volts require 0.4 amps. (Inter-Coll. Exam., Cambridge, 1908.)

Draw OE_1 the top part of the saturation curve.

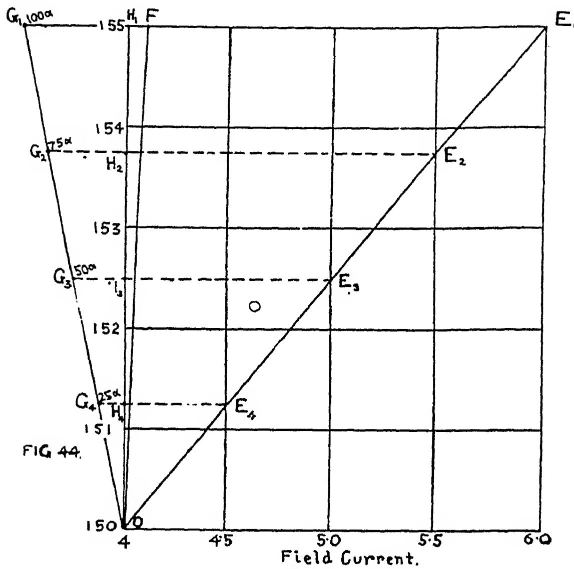


FIG. 44.

Now the lower part is straight, and such that 20 volts require 0·4 amps., and so 5 volts will require 0·1 amp. Draw OF corresponding to this ($H_1F = 0·1$, and $OH_1 = 5$ volts).

The drops corresponding to 100, 75, 50, 25, 0 amps. in the armature are 5, 3·75, 2·5, 1·25, 0 volts respectively. Set up these drops along OH_1, giving H_1, H_2, H_3, H_4, and O respectively.

Make $FG_1 = 0·6$ amps, and join OG_1.

Draw horizontal lines through H_2, H_3, H_4, cutting OG_1 in G_2, G_3, G_4, and OE_1 in E_2, E_3, E_4.

We see that the E.M.F.'s are 155, 153·75, 152·5, 151·25, and 150 volts.

The total field currents are

$$H_1G_1 + H_1E_1 = 6·5 \text{ amps.},$$

and so on for the others, giving 5·875, 5·25, 4·625, 4 amps.

The corresponding shunt resistances are

$$\frac{150}{6\cdot5}, \quad \frac{150}{5\cdot875}, \quad \frac{150}{5\cdot25}, \quad \frac{150}{4\cdot625}, \quad \frac{150}{4}$$

$$=23\cdot1, \ 25\cdot6, \ 28\cdot6, \ 32\cdot4, \ 37\cdot5 \text{ ohms.}$$

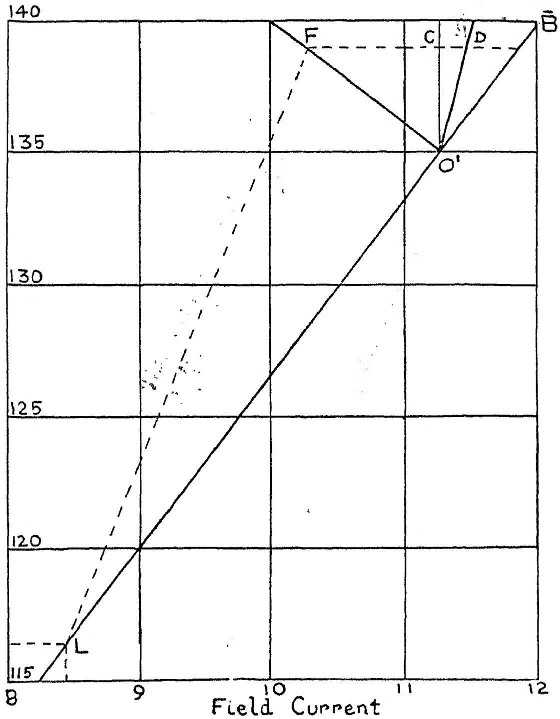


FIG. 45

Example 2. The data for a given dynamo are as follows :

Armature Resistance : 0·01 ohm.

Saturation Curve : E.M.F.	Field Current.	
0	0	straight.
20	1	
120	9	straight
140	12	

Short Circuit Curve : Armature Current.	Field Current.	
0	0	straight.
500	1·5	

If the no-load voltage be 135, find the terminal P.D. when the armature current is 400 amps., if the field resistance be kept constant at its no-load value. (Inter-Coll. Exam., Cambridge, 1910.)

At 500a, armature drop is 5 volts.

When E.M.F. = 135, the field current on no-load = 11·25 A.

∴ no load field resistance = 12 ohms.

Proceeding as in § 63, we read from the diagram (Fig. 45) that when the armature current is 400 A.

E.M.F. = 116·4 V.

the drop = O′C = 4 volts.

∴ P.D. = 112·4 V.

What is then the load ?

Field current = 8·44 + FC,
= 9·44 A.,

∴ external current = 400 − 9·4,
= 390·6 A.

Example 3. A shunt machine is to deliver about 440 k.w. at 550 volts. The saturation curve was given by :

Field Current	1	5	6	7	8	10	ampères
E.M.F.	120	510	550	580	605	623	volts.

The short-circuit current with 1·13 amps. excitation was 800 amps. The armature resistance was 0·025 ohms. Find the curve connecting output and added shunt resistance, if the shunt resistance at full load is that of the windings only. (Mech. Sc. Trip., 1909.)

The full load armature current $= \dfrac{440{,}000}{550} = 800$ A. With this armature current, the internal drop is 800 × ·025 = 20 volts, and if the P.D. = 550, the E.M.F. = 570. Hence we need only draw the saturation curve between 550 and 570 volts. We then proceed as in Ex. 2, p. 58, and read the following results from the diagram (Fig. 46) :

P.D. constant = 550 V.

Armature Current = 100, 200, 300, 400, 500, 600, 700, 800 amps.

Total Field Current = 6·78, 6·82, 6·85, 6·89, 6·93, 6·97, 7·0, 7·04.

External Current = 93·22, 193·18, 293·15, 393·11, 493·07, 593·03, 693, 792·96.

Shunt Resistance = 81·2, 80·8, 80·4, 79·9, 79·4, 79·0, 78·6, 78·2 ohms.

Shunt Resistance at full load = 78·2. = resistance of actual winding only.

Added Shunt Resistance = 3·0, 2·6, 2·2, 1·7, 1·2, 0·8, 0·4, —.

Output = 51·2, 106, 161, 216, 271, 326, 371, 436 K.W.

The last two rows give the values of outputs and the corresponding added shunt resistance.

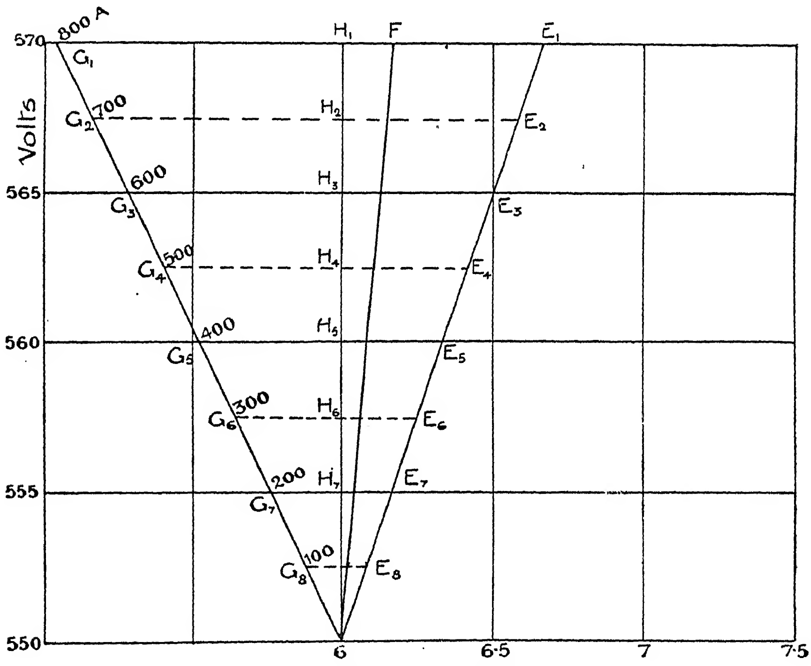


FIG. 46

§ 67. **The Self-Excitation of Dynamos.** When the field magnets have once been magnetized from an external source they retain a certain portion of their magnetism, and it is this remanent magnetism which enables a dynamo to excite itself

when started. As soon as the armature begins to turn, the coils, cutting the flux of the remanent magnetism, have a small E.M.F. induced in them, and this starts a small current flowing in the field winding, which increases the flux. The increase of flux causes an increase of E.M.F., and so a larger current flows and the flux is still further enlarged, and so on until the machine is fully excited.

Under certain conditions the dynamo may refuse to excite. For example, the initial E.M.F. may be too small owing to the speed being too low. If the field connections and the direction of rotation are not properly related the initial E.M.F. will set up a flux tending to destroy the remanent magnetism instead of strengthening it. If the resistance of the field circuit, or of the brush contacts, be too large, the initial current will be too small. These are the principal causes which may render self-excitation impossible.

§ 68. **The Conditions Necessary for Self-Excitation.** We now try to find an expression which must be satisfied if a *shunt* dynamo is to excite itself.

Let $\mathscr{R}_a$ = the reluctance of the armature.
$\mathscr{R}_s$ = ,, ,, ,, ,, field magnets.
$\mathscr{R}_g$ = ,, ,, ,, ,, air gap.
r_a = ,, resistance ,, ,, armature.
r_s = ,, ,, ,, ,, shunt coils.
R = ,, ,, ,, ,, external circuit.
N_a = ,, number of turns in series on the armature, between one pair of brushes.
N_s = ,, number of turns on the field windings.

Then, with the usual notation, we have

$$E = 4\Phi n N_a \cdot 10^{-8}.$$

$$= 4nN_a \cdot 10^{-8} \times \frac{\frac{4\pi}{10} I_s N_s}{\mathscr{R}_a + \mathscr{R}_s + \mathscr{R}_g}$$

$$= \frac{16\pi \cdot 10^{-9} . nN_a N_s I_s}{\mathscr{R}_a + \mathscr{R}_s + \mathscr{R}_g}$$

also it is easy to shew that

$$I_s = \frac{R}{Rr_s + r_s r_a + r_a R} E.$$

Substituting the above value for E, we have

$$\therefore \quad I_s = \frac{16\pi \cdot 10^{-9} \cdot nN_aN_sI_sR}{(Rr_s + r_sr_a + r_aR)(\mathcal{R}_a + \mathcal{R}_s + \mathcal{R}_g)}$$

$$\therefore \quad \mathcal{R}_a + \mathcal{R}_s + \mathcal{R}_g = \frac{16\pi 10^{-9}.\ nN_aN_s}{r_s + r_a + \dfrac{r_sr_a}{R}}$$

Hence, if the machine is to excite itself on open circuit ($R = \infty$) we must have

$$\mathcal{R}_a + \mathcal{R}_s + \mathcal{R}_g \overline{\overline{<}} \frac{16\pi.10^{-9}.\ nN_aN_s}{r_a + r_s}.$$

But in the initial stages of excitation the reluctance of the iron is negligible compared to that of the air-gap, and r_a is very small compared to r_s, so we can write for the condition that must be satisfied,

$$\mathcal{R}_g \overline{\overline{<}} 16\pi.10^{-9}.\frac{nN_aN_s}{r_s},$$

or

$$n \overline{\overline{>}} \frac{10^9.\ r_s\mathcal{R}_g}{16\pi\ N_aN_s} \quad \dots\dots\dots\dots\dots\dots \quad (27).$$

The latter equation gives the lowest speed at which the machine will excite itself.

It is to be noted that the part $\dfrac{\mathcal{R}_g}{N_aN_s}$ of the expression on the right can readily be obtained from the open circuit curve of the machine. For, neglecting $\mathcal{R}_a$ and $\mathcal{R}_s$, we have

$$E = \frac{16\pi.10^{-9}.\ nN_aN_sI_s}{\mathcal{R}_g},$$

hence

$$\frac{\mathcal{R}_g}{N_aN_s} = 16\pi.10^{-9}.\ n\frac{I_s}{E} \quad \dots\dots\dots\dots\dots\dots \quad (28).$$

In this equation n is the speed at which the open circuit test is made, and $\dfrac{I_s}{E}$ is obtained from the practically straight initial part of the curve.

The conditions for a *series*-wound dynamo may be obtained in a precisely similar manner. The condition that it may excite itself is

$$16\pi.10^{-9}.\ nN_aN_s = (r + R)\mathcal{R}_g.$$

where r = the resistance of the machine itself, and R = that of the external circuit.

If the machine is to excite on short circuit it is only necessary to put $R=0$ in the above. We then have the condition.

$$n \geqq \frac{10^9}{16\pi} \cdot \frac{r\mathcal{R}_g}{N_a N_s} \qquad \ldots\ldots\ldots\ldots (29).$$

§ 69. **Example.** A dynamo is run with separate excitation, at a constant speed of 500 R.P.M., on open circuit, and the E.M.F. measured as follows :

Field Current	0·5	1·0	1·5	2·0	2·5
E.M.F. ,,	40	80	100	110	115.

The resistance of the field coils is 60 ohms. Find what is the minimum speed at which the dynamo will excite itself when connected as a shunt wound machine. (Mech. Sc. Trip., 1914.)

$$\text{We have } \frac{\mathcal{R}_g}{N_a N_s} = 16\pi . 10^{-9} . \frac{500}{60} . \frac{I_s}{E} = 0{\cdot}419 . 10^{-6} \frac{I_s}{E}.$$

The value of $\frac{I_f}{E}$, from the open circuit curve, is, up to 80 volts, 0·0125.

$$\therefore \qquad \frac{\mathcal{R}_g}{N_a N_s} = 0{\cdot}419 \times 0{\cdot}0125 \times 10^{-6} = 0{\cdot}523 . 10^{-8}$$

Hence we must have

$$n \geqq \frac{r_s \mathcal{R}_g}{N_a N_s} \frac{10^9}{16\pi}$$

$$\geqq \frac{60 . 10^9}{16\pi} \times {\cdot}523 . 10^{-8}$$

$$\geqq 6{\cdot}25.$$

i.e. the minimum speed is 375 R.P.M.

COMPOUND WOUND GENERATORS.

§ 70. We have seen, in the preceding sections (§§ 61 and 62), that if the shunt resistance be kept constant, the E.M.F. and P.D. decrease as the armature current increases. When necessary the P.D. may be kept nearly constant by what is called

"compounding," that is by winding some of the field turns in series with the armature.

If the P.D. is to remain constant we must neutralize two effects: armature drop and armature reaction. With regard to the latter we have seen (§§ 62 and 65) that the effect of armature reaction is equivalent to a reduction of shunt current, by an amount which is proportional to the armature current so long as the brush lead is not altered. If, then, we wind on the fields a suitable number of turns in series with the armature, we can provide extra ampère-turns which will always balance the demagnetizing effect of the armature, the effects of armature reaction will be eliminated, and the curve of E.M.F. will be the same as the open-circuit characteristic.

With regard to the armature drop: since $V=E-I_a r_a$, we require that E shall increase in proportion to the increase of I_a. Over the working range we are usually only concerned with the top part of the open circuit curve, and this is nearly or quite straight, so that to obtain the requisite rise of E we require extra ampère-turns on the fields which shall increase in proportion to the rise of I_a. This can be effected by winding extra field turns in series with the armature. Thus, by employing the proper number of series turns we can keep the P.D. constant for all values of I_a, provided (i) the brush lead is not changed, and (ii) the open circuit characteristic is straight over the working range.

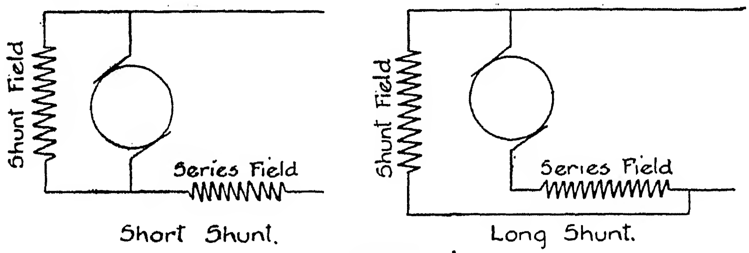


FIG. 47.

§ 71. **To find the number of series turns required.**

Let I_1 = shunt current at no-load for given P.D.

I_2 = ,, ,, ,, given load ,, ,,

I_a = armature ,, ,, ,, ,, ,,

N_1 = number of shunt turns.

N_2 = ,, ,, series turns required.

Then, the extra shunt current $=I_2-I_1$.

$\therefore$ „ ampère-turns required $=N_1(I_2-I_1)$.

These are to be provided by I_a flowing in the series turns, hence

$$N_2 I_a = N_1(I_2-I_1),$$

$$N_2 = \frac{N_1(I_2-I_1)}{I_a} \quad \ldots\ldots\ldots\ldots\ldots \quad (30).$$

§ 72. **Example 1.** By a short circuit test it is found that the demagnetizing effect of the armature of a shunt dynamo, when carrying 500 A. is balanced by 0·4 A. in the shunt. The resistance of the armature and series coils is 0·02 ohm. To give 225 volts on open circuit the field current is 4 A.; to give 200 volts it is 3 A. The shunt turns number 10,000. Find the series turns to compound correctly with 500 A. in the armature, the P.D. being 200 volts. Assume the saturation curve to be a straight line between the given points. (Inter-Coll. Exam., Cambridge, 1909.)

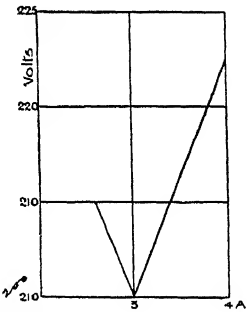


FIG. 48

This is so simple that it is hardly necessary to draw a figure. However, one is given to show how the method of working follows the preceding text.

At 500 A. the armature drops $=10$ volts.

$\therefore$ E must $=210$, if $V=200$ V.

At 210 volts, $I_s = 3 + \dfrac{4-3}{225-200} \times (210-200) = 3{\cdot}4$ A. . . (i).

Besides this we require 0·4 A. to balance armature reaction; thus the total $I_s = 3{\cdot}8$ A.

At no-load $I_s = 3$ A.

$\therefore$ at 500 A. the extra $I_s = 0{\cdot}8$ A.

Let N_2 = number of series turns required to provide this extra excitation, then

$$N_2 \times 500 = 0{\cdot}8 \times 10{,}000,$$

$$N_2 = \frac{8000}{500} = 16.$$

Of course (i) can be read off from the drawing, but it takes longer to make the figure than to do this easy proportion sum.

Example 2. The open circuit characteristic of a shunt-wound dynamo is defined by three points A, B, C:

	I_s	E	
A	0·6	60	straight to the origin.
B	3	220	straight.
C	4·8	235	

With the armature short-circuited a field current of 0·92 A. gives $I_a = 80$ A.

The armature resistance $= 0{\cdot}15$ ohm.

If the dynamo is to give 100 A. at 220 volts, what must be the shunt resistance? If the current is reduced to 40A., what increase of shunt resistance is necessary to maintain the P.D. at 220 volts? If the shunt winding has 4000 turns and a resistance of 73·3 ohms, show that the addition of 112 series turns of negligible resistance will keep the P.D. constant at 220 volts for currents ranging from 0 to 100 A. (Mech. Sc. Trip., 1911.)

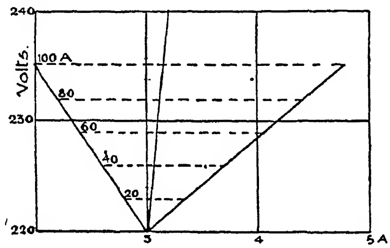


FIG 49

From the figure we find

$$\text{at } 100 \text{ A.},\ I_s = 5{\cdot}80,\ V = 220, \quad \therefore \quad r_s = 37{\cdot}8 \text{ ohms.}$$

$$40 \text{ A.},\ I_s = 4{\cdot}12,\ V = 220, \quad \therefore \quad r_s = 53{\cdot}4 \ \text{,,}$$

$$\text{extra } r_s = 15{\cdot}6 \text{ ohms.}$$

We see that 73·3 is the resistance at no-load.

From the diagram, the extra exciting current required above the no-load value

$$= 0{\cdot}01\, I_a \text{ (for reaction)} + 0{\cdot}018\, I_a \text{ (for armature drop)},$$
$$= 0{\cdot}028\, I_a.$$

Let N_2 = number series turns required, then

$$4000 \times 0{\cdot}028 I_a = N_2 \times I_a,$$
$$\therefore \quad N_2 = 112.$$

Hence 112 series turns provide the right amount of extra excitation, whatever the value of I_a.

OPERATION IN PARALLEL.

§ 73. In generating stations it is usual to have several machines connected to the same bus-bars. The advantages

of this are: (1) The number of machines actually in use at a given time corresponds with the demand, thus each is working at about its greatest efficiency. (2) If one machine breaks down, the whole station is not disabled; moreover, in large stations a single generator could not fulfil the demand.

§ 74. **Shunt Machines in Parallel.** A diagram of connections is shown in Fig. 50.

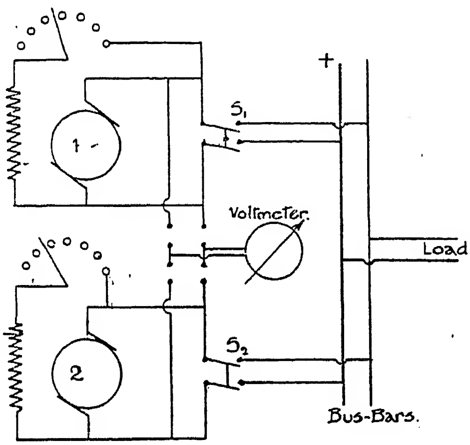


FIG. 50

Suppose generator (1) is working, and we wish to switch in (2): We excite (2) to give the same voltage as (1), and then close its main switch. Then (1) will still supply all the load unless its excitation be reduced, and that of (2) increased, for, directly (2) began to take any load its E.M.F. would fall and be less than the P.D. of (1), with the result that currents trying to flow out of (2) would be forced back. By regulating the field resistances the load can be divided at will. To shut down (1) we do not merely open its main switch, or there will be an arc, and a violent shock to (2), but we reduce its excitation, and increase that of (2) until the latter carries the whole load, and then open S_1. When the machines are running in parallel, each will take a share of the load, the proportions into which the total current is divided depending on their E.M.F.'s, and internal drops. The terminal P.D.'s of the two must be equal, and this, in calculations, is used to determine the ratio into which the current divides. If the two

machines are identical in every respect, equally excited, and running at exactly the same speeds, the load will be equally divided between them. But if the field current or the speed of one be reduced, its E.M.F. will decrease, and the portion of the load taken by this one will decrease, whilst the other will take more of the current; this will go on until the two P.D.'s are equal. When the E.M.F. of the first machine equals the P.D. of the other, the former will take no load at all; if the E.M.F. still decreases it will begin to take current from the other and run as a motor in the same direction (see § 78). The current taken tends to increase its speed, and consequently its E.M.F.

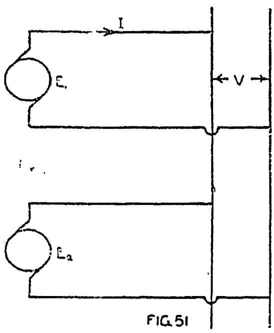


FIG. 51

§ 75. **Example 1.** A 30 *k*w. dynamo and a 20 *k*w. dynamo, whose armature resistances are 0·025 ohm and 0·03 ohm respectively, are running in parallel, giving the above outputs, on a circuit of 0·25 ohm resistance. If the shunt circuits be neglected, find the E.M.F. of each machine, and the current in each armature. (Inter-Coll. Exam., Cambridge, 1914.)

Let E_1 = E.M.F. of the 30 *k*w. machine.
,, E_2 = ,, ,, ,, 20 *k*w. ,,
,, V = common P.D.
,, I = current in the 30 *k*w. machine.

The total current $= \dfrac{V}{0\cdot25} = 4V$.

∴ The current in the 2nd machine $= 4V - I$.

Hence $V = E_1 - I \times 0\cdot025 = E_2 - (4V - I)0\cdot03$.

Also $VI = 30{,}000$ and $V(4V - I) = 20{,}000$.

These equations give $V = 112$ V.

$$\therefore \qquad I = \frac{30{,}000}{112} = 268 \text{ A.}$$

$$E_1 = 112 + \cdot025 \times 268 = 118\cdot7 \text{ V.}$$

$$E_2 = 112 + \cdot03 \times 180 = 117\cdot4 \text{ V.}$$

The current in the 20 *k*w. dynamo is $4V - I = 180$ A.

Example 2. Two shunt machines are working in parallel and share between them a load of 1400 A. Their characteristics, etc., are given below :—

		(I) I_s	(I) E	(II) I_s	(II) E
Saturation curves		0	0	0	0
		0·3	20	0·4	20
		3	200	4	200
		4·5	220	6	220
Short-circuit curves	I_s	0	0·96	0	1·28
	I_a	0	800	0	900
Armature resistance		0·04 ohm.		0·03 ohm.	
Shunt ,,		38·8 ,,		36·0 ,,	

The curves may be assumed straight over the range required. Find the current taken by each dynamo, and the common P.D.

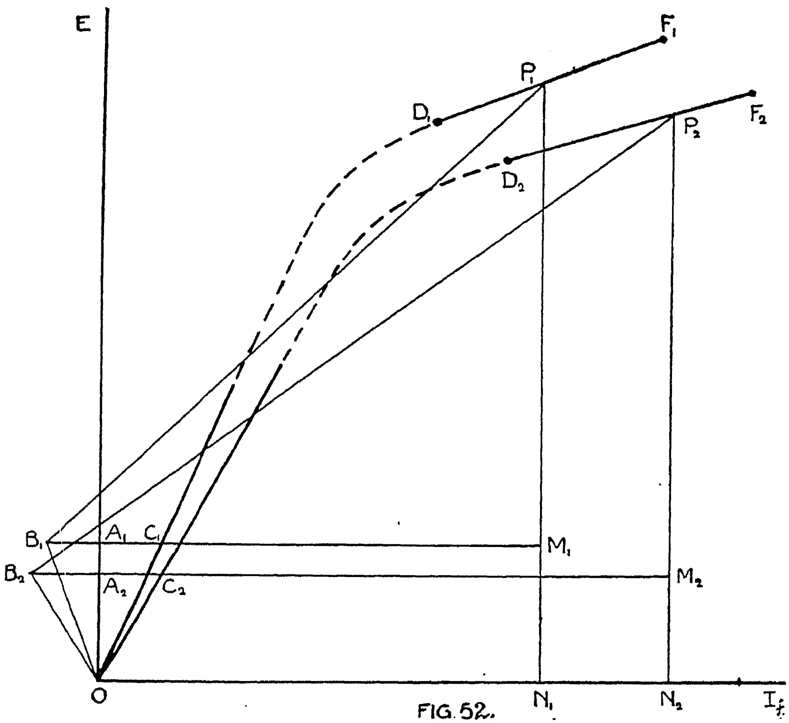


FIG. 52.

The diagram for the problem is sketched in Fig. 52. It cannot be drawn accurately as we do not know the armature

currents, and so cannot find the points B_1 and B_2. The figure is exactly the same as Fig. 42, and the lettering is the same, so that no explanation is necessary; it shows the curves for both dynamos. The problem is to make the P.D.'s of the two machines equal, i.e. to make $P_1M_1 = P_2M_2$, and in order to solve the problem we shall use the methods of coördinate geometry.

The equations of D_1F_1 and D_2F_2 are

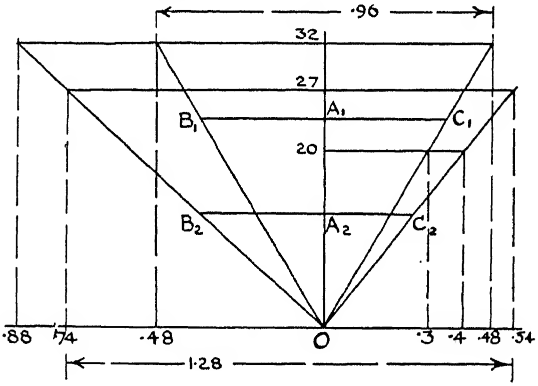
$$\frac{E-200}{220-200} = \frac{I_s-3}{4\cdot5-3} \text{ and } \frac{E-200}{220-200} = \frac{I_s-4}{6-4}$$

$$\left.\begin{array}{ll} \text{or} & 3E - 40I_s = 480 \\ \text{and} & E - 10I_s = 160 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots \text{(i).}$$

Let the current in (I) be I, then the current in (II) is $1400 - I$.

Hence $OA_1 = 0\cdot04 \times I$

and $OA_2 = 0\cdot03(1400 - I) = 42 - 0\cdot03I$.



Then (see Fig. 53).

$$A_1B_1 = \frac{OA_1}{32} \times 0\cdot48 = 0\cdot0006I$$

$$A_2B_2 = \frac{OA_2}{32} \times 0\cdot88$$

$$= 1\cdot16 - 0\cdot00083I.$$

Thus, the coördinates of B_1 and B_2 are

$\{-0\cdot0006I, 0\cdot04I\}$ and $\{-(1\cdot16 - 0\cdot00083I), (42 - 0\cdot03I)\}$

The inclinations of B_1P_1 and B_2P_2 (Fig. 52) are $\tan^{-1}38\cdot8$ and $\tan^{-1}36$. Hence their equations are

$$E - 0{\cdot}04I = 38{\cdot}8(I_s + {\cdot}0006I),$$

and $$E - ({\cdot}42 - 0{\cdot}03I) = 36(I_s - 0{\cdot}00083I + 1{\cdot}16),$$

or $$\left.\begin{aligned} E - 38{\cdot}8I_s &= 0{\cdot}0633I \\ E - 36I_s &= -0{\cdot}06I + 83{\cdot}7 \end{aligned}\right\} \qquad \text{(ii)}.$$
and

The intersections of (i) and (ii) give the points P_1 and P_2.

On solving these equations we find that

at P_1, $P_1N_1 = E_1 = 244 - 0{\cdot}0331I$,

and at P_2, $P_2N_2 = E_2 = 189 + 0{\cdot}0231I$.

Also $$\left.\begin{aligned} P_1M_1 &= P_1N_1 - OA_1 \\ P_2M_2 &= P_2N_2 - OA_2 \end{aligned}\right\} \qquad \text{(iii)}.$$
and

We must have $P_1M_1 = P_2M_2$. Hence, substituting in (iii) the values of the lengths found above, we obtain an equation for I:—

$$244 - 0{\cdot}0331I - 0{\cdot}04I = 189 + 0{\cdot}0231I - 42 + 0{\cdot}03I,$$

which gives $I = 770$ A.

Then the current in the other machine is $1400 - 770 = 630$ A.

The common P.D. is

$$\begin{aligned} V &= P_1M_1 \ (\text{or} = P_2M_2), \\ &= 244 - 0{\cdot}0331I - 0{\cdot}04I \\ &= 244 - {\cdot}0731 \times 770 \\ &= 188 \text{ V}. \end{aligned}$$

§ 76. **Compound Machines in Parallel.** The connections are the same as for shunt machines, except as regards the equalizing cable, which is necessary. Suppose A_1 is open (i.e. suppose there were no such cable), and that (2) gives an higher E.M.F. than (1), then (2) will take more of the load, and so the series field of (2) will be increased, and so its E.M.F. become even larger. This effect will be cumulative until (2) takes all the load, and (1) none, and after this the current in (1) will be reversed; and this, happening in the series coil, weakens the field, and so it runs faster. Then the increased speed causes a bigger E.M.F.; current rushes into the other machine, and the positions are reversed. The effect of the equalizing cable is to keep the currents in the series coils nearly equal, even though the armature currents differ widely, and so the above does not happen. For suppose, as before, that the E.M.F. of (2) is greater than

that of (1), and therefore (2) supplies more current than (1), there will be a bigger drop between B_2 and C_2 than between B_1 and C_1. If however we have an equalizing cable of low resistance, as shown, this cannot happen, and some of the current from (2) will flow through the equalizing cable and the series coil of (1), thus assisting (1) to keep up its pressure.

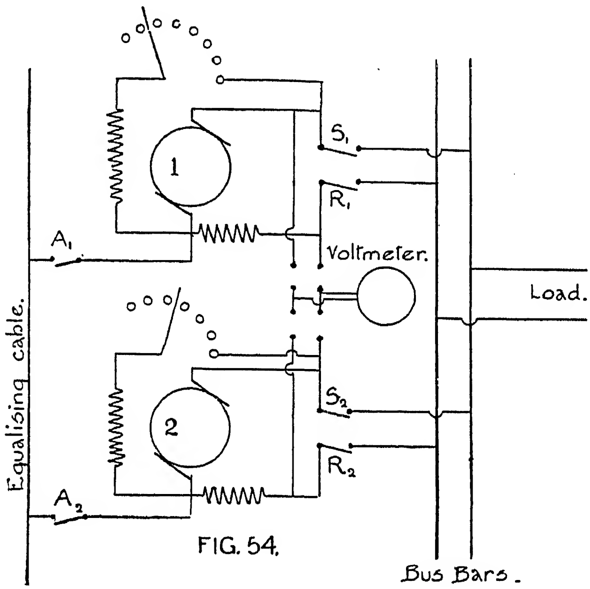


FIG. 54.

To connect two Compound Machines in Parallel. Suppose (1) is taking all the load, and we wish to connect (2) in parallel with it. We first excite (2) to give its full voltage, and close switches (4) and A_2, so that some of the current from (1) goes through the series coils of (2). Adjust the voltage of (2), and then close switch (3), and regulate the excitation so that both carry half the load. To disconnect (2) we perform the operations in the reverse order.

EXAMPLES.

1. A two-pole dynamo has a total magnetic flux per pole of $3 \cdot 5 \cdot 10^6$ lines. The armature has 240 peripheral conductors. Find the speed at which it must be run to generate 108 volts. (Engineering Special, Cambridge, 1908.)

2. A six-pole wave-wound dynamo has 304 peripheral wires on the armature, and the E.M.F. is to be 550 volts at 720 R.P.M. Find the flux per pole.

3. A 550 volt machine has 8 poles and runs at 500 R.P.M. The average volts per commutator segment are not to exceed 11. Each armature coil is to contain one turn only, and the number of commutator segments is to be divisible by 3. What must be the magnetic flux per pole?

4. A four-pole D.C. dynamo runs at 900 R.P.M.; the flux per pole is $12 \cdot 10^5$ lines, and the armature winding is two-circuit. The total number of armature wires is 620. Calculate the E.M.F. generated.

5. A 200 K.W. generator is to give 500 volts at 400 R.P.M., and has eight poles. The armature is to be lap wound, and the flux per pole is $45 \cdot 10^5$. Find the number of armature wires necessary.

6. A two-pole machine has the following dimensions:

Field and yoke: length of iron = 120 cm., mean area of section = 200 cm^2.
Armature: length of iron path = 50, mean section = 220 cm^2.
Air-gap = 0·6 cm. each. Area of pole face = 500 cm^2.
Coefficient of dispersion = 1·2. Speed = 1200 R.P.M.
Armature turns = 500. E.M.F. = 250 volts.
Find the ampère-turns necessary on the fields.
Use iron curves given for Lohys iron on p. 113.

7. Given the following dimensions for the magnetic circuit of an eight-pole dynamo, determine the number of ampère-turns required at full load:—

		Mean Length cms.	Mean Section cm$_2$.	μ
Armature core	steel	85 (pole to pole)	1190	1850
,, teeth	,,	3·3	1065 (area under pole face)	42
Air gap	air	0·9	2440 (area under pole face)	1
Pole Core	steel	50	1740	338
Yoke	,,	145 (pole to pole)	1390	755

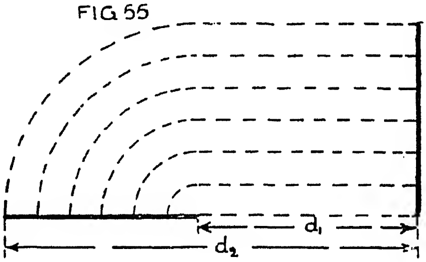
Coefficient of dispersion 1·2.
P.D. =550 volts. Armature current =730 amps.
Resistance of armature =0·026 ohm.
Speed =100 R.P.M. Lap-wound armature.
Number of conductors =1584. (R.N.C., Greenwich, 1912-13.)

8. Shew that the permeance of the leakage path shown in Fig. 55 is

$$\frac{2a}{\pi}\log_e\left\{(d_2-d_1)+\frac{2}{\pi}d_1\right\},$$

where a is the length of the surfaces perpendicular to the plane of the paper.

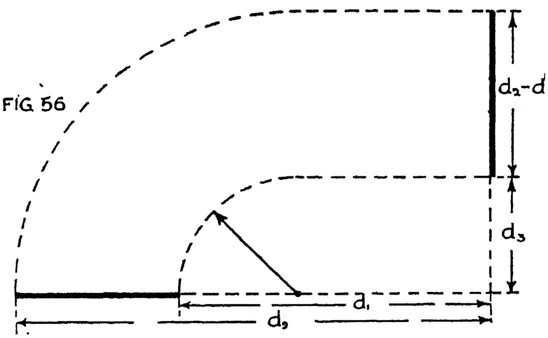
FIG 55



9. Show that the permeance of the leakage path shown in Fig. 56 is

$$\frac{2a}{\pi}\log_e\left\{\frac{d_2+d_3-d_1+\frac{2}{\pi}(d_1-d_3)}{d_3+\frac{2}{\pi}(d_1-d_3)}\right\}$$

FIG 56



10. A two-pole dynamo has an output of 200 *k*w. at 250 v.; there are 200 conductors on the armature and the lead of the brushes is 27°. Find the number of demagnetizing ampère-turns. (Inter-Coll. Exam., Cambridge, 1903.)

11. An eight-pole dynamo has a full load armature current of 730 A. The number of conductors is 1584, and the lead is 4°. The armature is lap-wound. Find the number of demagnetizing ampère-turns.

12. Calculate the demagnetizing ampère-turns for the machine described on p. 31, if the brushes be placed at the polar horns, and the cross magnetizing effect be neglected.

13. Some particulars of a six-pole D.C. generator are as follows:—

Speed .... 750 R.P.M.
Armature,
  Winding .... single, lap.
  No. of conductors .... 240.
  Diameter at air-gap .... 53·5 cm.
  Gross length .... 25·5 cm.
  3 Air-ducts, each 1·5 cm. wide.
Commutator,
  No. of segments .... 120.
  Diameter .... 42·3 cm.
Brushes, width .... 7/8″.

Calculate the Reactance Voltage according to Hobart's method when the current in the external circuit is 1000 ampères. (R.N.C., Greenwich, 1911-12.)

14. The particulars of a certain generator are as follows:

Number of poles .... 12.
Number of armature conductors ...... 1392.
Number of commutator segments .... 696.
E.M.F. at full load .... 600 V.
Speed, R.P.M. .... 75.
Full-load current .... 2500 A.
Length of armature conductors in slot .. 100 cm.

The conductors are straight bars, one in each slot, and are connected to form a 12-circuit single drum winding. The brushes are twice the width of a commutator segment. It may be assumed that the flux linked with each conductor in a slot is 10 lines per cm. length per ampère, and that the resistance of the armature conductor is negligible. Calculate the necessary strength of the commutating field, using Hobart's method. (Mech. Sc. Trip. 1908, B.)

15. A dynamo's saturation curve gave 500 V. with 4 A. shunt current, 550 V. with 6 A., and 100 volts with 0·4 A. The armature resistance was $\frac{1}{4}$ ohm. The short circuit curve gave 100 A. armature current with $\frac{1}{2}$ A. in the field. When the

armature current is 80 A. and the P.D. is 500 V., find the shunt resistance.

16. The following observations were taken on a shunt dynamo: Resistance of fields, 25 ohms; resistance of armature, 0·1 ohm; with a field current of 3 A. the E.M.F. was 97 V., with 4 A. it was 100 V., with 0·2 A. it was 8 V.; a short circuit current of 80 A. required 0·4 A. in the field.

Assuming the saturation curve straight from 97 volts and beyond, determine the terminal P.D. when the machine is carrying 80 A. in its armature, assuming the lead the same as in the test. Also determine the output in kilowatts. (Inter-Coll. Exam., Cambridge, 1906.)

17. A dynamo had the following saturation curve:

Field current	0·25	3·5	4	4·5	5
E.M.F.	12·5	118	123	127	130.

The short circuit test gave 100 A. in the armature with 0·6 A. in the field. The armature resistance was 0·05 ohm, and the shunt resistance 27 ohms. Find the P.D. and the load with an armature current of 100 A. (Inter-Coll. Exam., Cambridge, 1911.)

18. A series dynamo has the following saturation curve and short-circuit characteristic:—

E	I_s		I_a	I_s	
0	0	straight.	0	0	straight.
15	1		20	5	
80	15	straight.			
110	25				

Find the P.D. at the terminals when running with a load of 20 amps., if the armature resistance be 0·5 ohm, and the series coil resistance 0·25 ohm. (Inter-Coll. Exam., Cambridge, 1914.)

19. A certain machine required an exciting current of 0·1 A. to give an E.M.F. of 5 volts, 5 A. to give 140 V., and 7 A. to give 150 V., and the saturation curve was nearly straight between these points. It required 0·5 A. to give a short-circuit current of 100 A. in the armature. The armature resistance was 0·05 ohm; the shunt turns numbered 5000. Find the series turns required to compound at 140 volts, up to 100 A. in the armature. (Mech. Sc. Trip., 1910.)

20. Two similar machines, excited to give E.M.F.'s of 115 V. and 112 V. respectively, have armature resistances of 0·01 ohm each. If they are delivering a total current of 600 A., find the common P.D., and the current flowing in each machine The shunt currents may be neglected.

21. Two shunt dynamos share between them a load of 3000 A. Their particulars are:—

	I.		II.	
	I_f	E	I_f	E
Saturation curves	0	0	0	0
	0·5	100	0·5	60
	3	400	4	400
	5	450	6·5	450
Short circuit curves $I_f =$	0	0·55	0	0·85
I_a	0	960	0	1500
Armature resistance	0·042 ohm.		0·032 ohm.	
Shunt resistance	80 ohms.		68 ohms.	

Assuming the curves straight over the necessary ranges, determine how the current is divided between the two machines.

22. Find the minimum speed for automatic excitation of each of the machines given in Questions 16-19, if the speed of the tests was 500 R.P.M.

CHAPTER III.

DIRECT-CURRENT MOTORS.

PRINCIPLES OF D.-C. MOTORS.

§ 77. A direct-current electric motor is constructed on exactly the same principle as a D.-C. generator, and may be either series, shunt, or compound wound. In fact, we may take a given machine, and either drive it by an engine, when it will deliver electrical power, or supply it with electrical power, when it will do mechanical work. The machine is, in fact, a medium for converting electrical energy into mechanical energy, or vice versa. Thus, much that we have said already applies equally to a generator or a motor.

§ 78. **Direction of Rotation of Motors.** With the poles of the field magnets, and the direction of the current through the armature, as shown in Fig. 57, by the right-handed screw law, we see that there will be a force on each conductor which tends to turn the armature round in an anti-clockwise direction. Now suppose we drive it as a dynamo (cf. p. 18); in order that the induced currents may be in the direction shown, we must drive the machine in the other direction. In other words, for a given direction of currents, there is a certain force between the field magnets and the conductors ; in one case we supply power to drive the machine against these forces, and in the other case we let these forces have their own way. Thus for a given

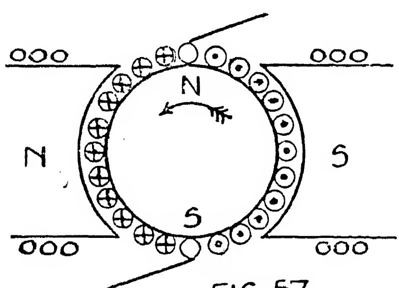


FIG 57

direction of current in the armature and field windings, the armature will rotate in one direction as a motor, and in the opposite direction as a dynamo. To reverse the direction of rotation we must reverse the current in the armature or the fields, but not in both.

§ 79. **Back E.M.F.** When an electric motor is running, the armature conductors are cutting flux, and therefore there will be an E.M.F. induced in them ; in fact the motor is driving itself as a generator. But, with the current going through the armature in a given direction, the armature must rotate one way as a motor and the other way as a dynamo; therefore, if it be made to rotate in the same direction in both cases, the currents will be in opposite directions, and also will be the E.M.F.'s which push the currents. Thus, when the motor is running, the E.M.F. induced in the armature wires by their rotation opposes that which is already sending current through them. This induced E.M.F. is called the *back e.m.f.*

§ 80. **The Gross Torque developed by a Motor.** We shall now find a formula for the theoretical torque of a motor, neglecting all sources of loss except the losses due to heating the coils.

Let V = the P.D. across the brushes of the motor.
E = the back E.M.F.
I_a = the armature current.
R_a = the armature resistance.

Then the applied P.D. has to overcome the induced E.M.F. and the resistance of the armature. Hence we must have

$$V = E + I_a R_a \quad \ldots\ldots\ldots\ldots\ldots\ldots (31).$$

This is the fundamental equation for a D.-C. motor. If we multiply this by I_a we have

$$VI_a = EI_a + I_a^2 R_a.$$

Of the three terms in this equation, VI_a is the total electrical power supplied to the armature, and of this an amount $I_a^2 R_a$ is wasted in heating the armature conductors. The remainder, EI_a, is converted partly into useful power and is partly wasted on losses in the iron due to hysteresis and eddy currents, and on friction. The value of the torque which we find from EI_a is called the gross torque of the motor.

Let M lbs.ft. = the gross torque.

n revs. per second = the speed of the motor.

Then the corresponding horse-power is $\frac{2\pi nM}{550}$, and this must be equal to $\frac{EI_a}{746}$. We have, then,

$$\frac{2\pi nM}{550} = \frac{EI_a}{746},$$

from which we find

$$M = 0.117\frac{EI_a}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32).$$

If we substitute in this the value of E from (17) we have

$$M = 0{\cdot}117(4\Phi Np10^{-8})I_a \quad \ldots\ldots\ldots\ldots (33),$$

which may be written

$$M = 0{\cdot}117\Psi I_a, \quad \ldots\ldots\ldots\ldots\ldots\ldots (34),$$

where $\Psi = E/n = 4\Phi Np \,.\, 10^{-8}$, and is called the *Induction Factor.*

§ 80. **Example.** Find the gross torque of the following four-pole motor: Length of armature 10″, diameter 18″, 240 conductors, each carrying 150 ampères, the polar arcs cover 66% of the circumference; the induction under the poles = 6000 lines/cm². (Birmingham, 1911.)

The circumference of the armature = 18π inches.

$\therefore$ the area of each pole face $= \frac{0{\cdot}66 \times 18\pi \times 10 \times (2{\cdot}54)^2}{4}$,

$= 600$ cm².

$\therefore$ Total flux from one pole $= 600 \times 6000 = 3{\cdot}6 \,.\, 10^6$ lines.

Torque in lbs. ft. (*mzI, p.* 25) $= {\cdot}117(p\Phi z \,.\, 10^{-8})I_a$,

where $p = 1$ for lap-wound armatures, and = 2 in this case if the armature be wave-wound. If the armature be lap-wound there will be four parallel circuits, so that the total current will be 600 ampères, i.e. $pI_a = 600$. For a wave-wound armature there will be only two circuits, so that $I_a = 300$ ampères, but $p = 2$, so that pI_a again = 600. Hence, in either case, we have

$$M = 0{\cdot}117 \times 3{\cdot}6 \,.\, 10^6 \times 240 \times 600 \times 10^{-8},$$

$$= 608 \text{ lbs.ft.}$$

§ 81. **Torque, Speed, and Current Relations.** We shall now consider the question: What controls the current and speed of a motor? For this purpose we must keep before us the following relations:

$$M = 0{\cdot}117 C\Phi I_a, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33a),$$

where $$C = 4Np \,.\, 10^{-8} = \frac{\Psi}{\Phi}, \text{ and is constant.}$$

$$I_a = \frac{V - E}{R_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35).$$

and $$E = C\Phi n. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36).$$

First, consider the armature current: From (33a) we see that the torque depends upon the flux and the armature current; in a shunt motor the flux is constant, for a given P.D., except for armature reaction, whilst in a series motor the flux is a function of the armature current and increases with it. Thus, in either case, the torque depends only on the armature current. Now suppose the armature takes more current than is necessary to provide the torque required to drive the load, the torque will be too large, and the motor will accelerate. The effect of this will be that the back E.M.F. will increase and cause the armature current to fall (by (35) above); suppose this continues until the armature current is too small to provide the requisite torque; then the speed will fall and the induced E.M.F. will fall too until a large enough current can flow to provide the necessary torque. Thus we see that the armature current automatically adjusts itself to the load.

Next, with regard to speed: For a given load we have seen that the armature current, I_a, is fixed, V is also fixed, and therefore, from (35), so long as R_a remains constant, the back E.M.F. must be constant. If, however, we put extra resistance into the armature circuit, the back E.M.F. must decrease in order to let the same current flow, and this can only happen by the speed falling if the flux is constant. Conversely, if we can decrease the resistance of the armature circuit, we shall bring about an increase of speed. Thus, for a fixed applied P.D., torque, and brush lead, the current is fixed, and the speed depends only on the resistance of the armature circuit, the speed rising in proportion as the resistance is decreased, and vice versa.

When a motor starts the back E.M.F. is very small until the speed rises, and if no extra resistance were put into the armature circuit the current would be very large and burn out the winding. For this reason a motor must never be started without extra resistance in series with the armature. (See § 84.)

We must now discriminate between the behaviour of a shunt-wound motor and that of a series-wound motor.

SHUNT MOTORS.

§ 82. The connections for running a shunt motor are shown in Fig. 58, in which it will be seen that the arc of the contact piece B is made larger than the arc of the resistance stops C, so that the lever A touches B and completes the field circuit before any current passes through the armature. Without this provision, at starting, the armature would take all the current and probably be burnt out; unless the flux were established before rotation began, there would be no induced back E.M.F. to bring down the current from its large momentary starting value.

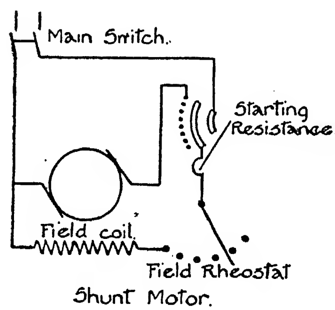


FIG. 58.

§ 83. **Speed of Shunt Motors.** Since the field is connected across the mains, the excitation remains constant for all loads if the field resistance be kept constant, neglecting the effect of armature reaction. Therefore, by equation (36) the speed varies directly as the back E.M.F.

Now $E = V - I_a R_a$, and we see that the principal and immediate cause of speed fluctuation, with varying load, in a shunt motor is the variation of the armature current and the consequent armature drop. But, as R_a is always very small, this variation is not great, perhaps ten per cent., from zero to full-rated load.

Armature reaction also has an effect on the speed, by causing a weakening of the main field. Since the back E.M.F.,

E, is determined by the equation $E = V - I_a R_a$, and is proportional to the product of flux and speed, any weakening of the flux will cause an increase of speed for a given armature current. Hence armature reaction may neutralize the effect of resistance drop, and so help to keep the speed constant, or even increase it above the normal.

The effect of armature reaction may be calculated from a saturation curve and the short circuit test.

Temperature changes affect the speed of a shunt motor, owing to the increase of resistance with rising temperature. The heating of the armature has only a very slight influence on the speed, for the resistance is small under any circumstances, so that the alteration of armature drop by heating has only a small effect upon the back E.M.F. The difference between the speeds with hot armature and cold armature may amount to about 0·5 per cent. of either.

On the other hand the heating of the field coils has an appreciable affect on the speed: with a temperature rise of 50°, the resistance of the winding will increase about 20 per cent., so that the field current will be reduced appreciably, the corresponding decrease of flux amounting to, perhaps, four or five per cent., and the speed increases by an equal percentage, other things being equal. This is an objectionable feature when a far greater approximation to constant speed is required; for example, in weaving. The difficulty can be overcome by using a field so highly saturated that the flux variation is small compared with the change of exciting current, but it is costly.

Example 1. A 500 V. shunt motor, running light at 600 R.P.M., takes 8 A. The field resistance is 100 ω, and the armature resistance 0·5 ω. Find the efficiency and the speed when the motor is giving 50 B.H.P.

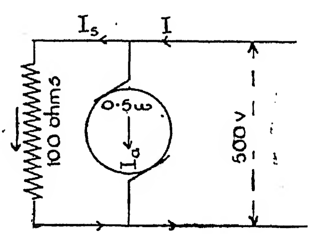


FIG. 59

As in § 80, we have the equation:

$$VI_a = EI_a + I_a^2 R_a. \qquad \text{(i).}$$

At no-load, EI_i represents the rotational losses of the armature, due to hysteresis, friction, etc. In this case we have

$I_s = 500 \div 100 = 5$ A., and $I_a = I - I_s = 8 - 5 = 3$ A. Hence, from (i), we have

$$500 \times 3 = \text{the rotational losses} + 3^2 \times 0{\cdot}5,$$

$\therefore$ the rotational losses $= 1500 - 4{\cdot}5 = 1495{\cdot}5$ W.

Since the speed of a shunt motor is nearly constant we can take this as constant.

$\therefore$ when giving 50 H.P., $EI_a = 50 \times 746 + 1495{\cdot}5$.

$\therefore$ from (i) $500 I_a = 50 \times 746 + 1495{\cdot}5 + 0{\cdot}5 I_a^2$,

or $$I_a^2 - 1000 I_a + 77{,}591 = 0,$$

which gives $I_a = 915$ or 85 A. Since I_a rises from zero when the motor is started, the lower value will be reached first, and, since it satisfies the electrical and mechanical equations, is the actual value at which the current will settle.

The total current, I, $= 85 + I_s = 90$ A.

$\therefore$ the total power supplied $= 90 \times 500 = 45{,}000$ W., and the output $= 50 \times 746 = 37{,}300$ W.

$$\therefore \quad \text{the efficiency} = \frac{37{,}300}{45{,}000} \times 100 = 82{\cdot}7\%.$$

At no-load $\quad E = 500 - 0{\cdot}5 \times 3 = 498{\cdot}5$ V.

At 50 H.P. $\quad E = 500 - 0{\cdot}5 \times 85 = 457{\cdot}5$ V.

Since the field current is constant, the flux is constant, if we neglect armature reaction and the back E.M.F. is directly proportional to the speed. At no-load the speed is 600 R.P.M., and therefore at 50 H.P. the speed is

$$\frac{457{\cdot}5}{498{\cdot}5} \times 600 = \text{about } 550 \text{ R.P.M.}$$

Example 2. A shunt motor developing 5 H.P. at 1000 R.P.M. is supplied with current at a pressure of 120 volts. If the armature current is 50 amps. at 5 H.P., and may be allowed to reach 75 amps., when starting, find the gross torque exerted by the motor when it starts, neglecting armature reaction.

If the torque corresponding with 5 H.P. at 1000 R.P.M. $= M$ we have:

$$2\pi . 1000 . M = 5 \times 33{,}000,$$

$$\therefore \quad M = \frac{5 \times 33{,}000}{2\pi \times 1000} = \frac{165}{2\pi} \text{lbs. ft.}$$

The field flux is constant because it is a shunt motor, neglecting armature reaction, and therefore we can write:

$$M \propto I_a.$$

At 5 H.P. $\qquad M = \frac{165}{2\pi}$ lbs. ft., and $I_a = 50$ amps.

At starting $\qquad I_a = 75$ amps.

$\therefore$ starting torque $= \frac{75}{50} \times \frac{165}{2\pi}$

$= 39 \cdot 4$ lbs. ft.

Example 3. A shunt motor with constant excitation runs light with 5 ampères in its armature. It runs at 1200 R.P.M. when loaded, taking 50 amps. in the armature at 100 volts P.D., and its armature resistance is 0·08 ohm. Find the nett shaft torque. (Inter-Coll. Exam., Cambridge, 1910.)

We have:

$$VI_a = EI_a + I_a^2 R_a,$$

i.e. $VI_a =$ rotational loss + output + $I_a^2 R_a$.

Running light we have:

$$100 \times 5 = \text{rotational loss} + 5^2 \times 0 \cdot 08.$$

$\therefore$ the rotational losses $= 500 - 2 = 498$ watts. We assume this to be constant.

Again, when $I_a = 50$, we have

$$100 \times 50 = 498 + \text{output} + 50^2 \times 0 \cdot 08.$$

$\therefore$ output $= 4302$ watts.

$= \frac{4302}{746}$ H.P.

If M = the torque, then

$$\frac{2\pi M 1200}{33{,}000} = \frac{4302}{746}$$

whence

$$M = \frac{4302}{746} \times \frac{33{,}000}{2\pi \times 1200}$$

$= 25 \cdot 3$ lbs. ft.

This is the nett shaft torque, i.e. the gross torque reduced by the amount required to overcome friction, etc.

§ 84. **Starters for Shunt Motors.** As mentioned in § 81, in starting a shunt motor it is necessary to prevent too large a current passing through the armature and damaging it, for, when at rest the armature develops no back E.M.F., and consequently the current through the armature would rise to a very high value, owing to the small resistance; to prevent this, some resistance is put in series with the armature and gradually cut out as the motor speeds up. This resistance is called a "starter" or "starting box."

§ 85. **Calculating the Number of Steps in a Starter.** Fig. 60 shows the armature circuit with starting resistances. Suppose there are n steps in the starter, the resistances between each successive pair of contact points being $r_1, r_2 \ldots . r_n$. Let R_a be the resistance of the armature.

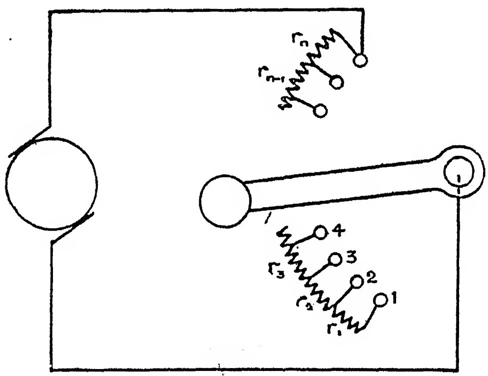


FIG 60

Let I_1 be the maximum current allowed at starting. Then, before the armature begins to move, E is zero, and, with the contact lever on stop 1, we have

$$I_1 = \frac{V}{(r_1 + r_2 + \ldots . + r_n) + R_a} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}.$$

Let I_2 be the minimum current for starting; then the armature will speed up until E is of such a value E_1 that the current becomes I_2, when we have

$$I_2 = \frac{V - E_1}{(r_1 + r_2 + \ldots . + r_n) + R_a} \quad \ldots\ldots\ldots\ldots \text{(ii)}.$$

Now the lever is moved to stop 2, thereby cutting out the resistance r_1, and the current increases, but falls again to I_2 as soon as E has become large enough owing to the increase of speed. If the same maximum current is reached, we have, for the *first moment* of contact with stop 2,

$$I_1 = \frac{V - E_1}{(r_2 + r_3 + \ldots . + r_n) + R_a} \quad \ldots\ldots\ldots\ldots \text{(iii)}$$

the back E.M.F. having, for the moment, still the same value. From these two equations it follows

$$\frac{I_1}{I_2}=\frac{(r_1+r_2+\ldots.+r_n)+R_a}{(r_2+r_3+\ldots.+r_n)+R_a}.$$

Similarly

$$\frac{I_1}{I_2}=\frac{(r_2+r_3+\ldots.+r_n)+R_a}{(r_3+r_4+\ldots.+r_n)+R_a}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{I_1}{I_2}=\frac{r_n+R_a}{R_a}$$

$$\therefore \quad \left(\frac{I_1}{I_2}\right)^n=\frac{(r_1+r_2+\ldots.+r_n)+R_a}{R_a},$$

but, by (i),

$$(r_1+r_2+\ldots.+r_n)=\frac{V}{I_1}-R_a$$

$$\therefore \quad \left(\frac{I_1}{I_2}\right)^n=\frac{\frac{V}{I_1}}{R_a}=\frac{V}{I_1R_a}$$

$$\therefore \quad n=\frac{\log\frac{V}{I_1R_a}}{\log\frac{I_1}{I_2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (37).$$

I_1 and I_2 being the upper and lower limits to the current as the lever is moved across the stops. This equation enables us to find the number of steps necessary.

§ 86. **To Find the Resistance of each Step.**

We have, from (ii) above,

$$E_1=V-(R+R_a)I_2$$

where R is written for $r_1+r_2\ldots.+r_n$

Also, from (iii),

$$R+R_a-r_1=\frac{V-E_1}{I_1}.$$

Eliminating E_1 from these two equations, we obtain

$$r_1=(R+R_a)\left(1-\frac{I_2}{I_1}\right)\ldots\ldots\ldots\ldots\ldots\ldots(38).$$

Again, on the next stop, we have

$$I_2 = \frac{V - E_2}{R + R_a - r_1}$$

and
$$I_1 = \frac{V - E_2}{R + R_a - r_1 - r_2}$$

whence, eliminating E_2,

$$r_2 = r_1 \frac{I_2}{I_1}.$$

Similarly

$$r_3 = r_2 \frac{I_2}{I_1},\ r_4 = r_3 \frac{I_2}{I_1} \text{ and so on,}$$

and
$$r_n = r_1 \left(\frac{I_2}{I_1}\right)^{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (39).$$

§ 87. **Example.** Find the number of steps and the resistance of each step of the starter to suit the following case :—

Pressure of mains........................440 volts.

Full-load current .... 60 amps.

Resistance of armature ....0·23 ohm.

Maximum value of starting current not to exceed 90 amps.

Minimum value of starting current to be exactly 60 amps. (R.N.C., Greenwich, 1911.)

The number of steps is given by

$$n = \frac{\log \dfrac{V}{I_1 R_a}}{\log \dfrac{I_1}{I_2}} = \frac{\log \dfrac{440}{90 \times \cdot 23}}{\log \dfrac{90}{60}}$$

$$= \frac{\log 21\cdot 25}{\log 1\cdot 5} = \frac{1\cdot 3273}{0\cdot 1661} = 7\cdot 54.$$

Obviously we must take the next greater whole number, viz. 8, as the value of w.

The whole resistance required $= \dfrac{440}{90} - 0\cdot 23,$

$= 4\cdot 66$ ohms.

We have, then,

$$\frac{I_2}{I_1} = 0{\cdot}666$$

$$R+R_a=4{\cdot}89,$$

whence

$$\begin{aligned}
r_1 &= 4{\cdot}89(1-{\cdot}666) &&= 1{\cdot}62 \text{ ohms.}\\
r_2 &= 1{\cdot}62\times{\cdot}666 &&= 1{\cdot}08 \quad ,,\\
r_3 &= 1{\cdot}09\times{\cdot}666 &&= 0{\cdot}72 \quad ,,\\
r_4 &= 0{\cdot}72\times{\cdot}666 &&= 0{\cdot}48 \quad ,,\\
r_5 &= 0{\cdot}48\times{\cdot}666 &&= 0{\cdot}32 \quad ,,\\
r_6 &= 0{\cdot}32\times{\cdot}666 &&= 0{\cdot}21 \quad ,,\\
r_7 &= 0{\cdot}21\times{\cdot}666 &&= 0{\cdot}14 \quad ,,\\
r_8 &= 0{\cdot}14\times{\cdot}666 &&= 0{\cdot}09 \quad ,,\\
\hline
R &&&= 4{\cdot}66 \quad ,,
\end{aligned}$$

§ 88. **Speed Control of Shunt Motors.**

(i) **By Armature Rheostat.** If the resistance drop in the armature circuit is increased, the back E.M.F., and therefore the speed, must fall, but we cannot *increase* the speed of the motor by this means. The chief objection is the enormous waste of power at low speeds, for, with a given torque and hence (§ 81) a given current, the same power is supplied all the time, but at low speeds a large portion of it is wasted as heat. A further disadvantage is that the fluctuation of speed with changes of load may be very great.

(ii) **By Variation of Field Current.** Since E is proportional to Φn, n varies inversely as Φ if E remain constant; ordinarily shunt motors are designed to work at such high inductions that the flux cannot be greatly increased, but by inserting resistance in the field circuit the strength of the field may be reduced, and so the speed is increased. Motors controlled by this means are constant output motors, not constant torque motors, for we have, approximately,

$$\text{H.P. output} \propto n \times \text{torque} \propto EI_a,$$

and

$$\text{torque} \propto \Phi I_a;$$

if I_a remain constant, E will be constant also, and so the output is constant; but, as the speed is increased by a weakening of Φ, the torque must be reduced, both by reason of the output being unchanged and by the second of the above equations.

For this and other reasons, it is not practicable to obtain more than a 20 or 30 per cent. increase of speed by this means, without some modification of the design of the motor.

(iii) **Multiple Voltage Control.** The power is supplied on a multiple-wire system (see p. 173), and the motor has a controller enabling the armature to be worked on any of the voltages which the system of supply permits. Thus, if the motor be connected to a three-wire system enabling pressures of 110 or 220 volts to be used, we have at once two speeds at which the motor will run, the ratio being about 1 : 2, and intermediate speeds can be obtained by an armature rheostat or field weakening.

The various systems of supply will be dealt with in Chapter VI.

§ 89. **Field Rheostats for Shunt Motors.** In order to decrease the speed below the normal, resistance is put in the armature circuit, but to increase the speed above the normal the field must be weakened, and this is effected by a regulating resistance placed in series with the field coils. In order to calculate what resistance is necessary for any given speed we proceed as follows: Let AB (Fig. 61) be the open circuit curve

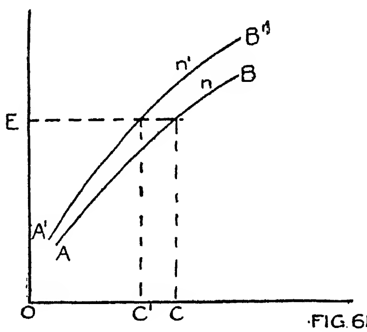


FIG. 61

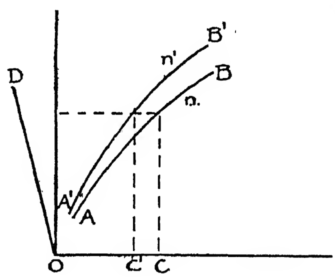


FIG. 62.

of the machine when run separately excited as a generator at the normal speed n, then the open circuit curve for a new speed n^1 can be found from the first by multiplying all the ordinates of the latter by n^1/n. Let this new curve be A^1B^1; then we see that, in order to produce the same back E.M.F., OE, at the new speed n^1, the exciting current has to be OC^1, neglecting the

effect of armature reaction. The total resistance of the field circuit must be $\frac{V}{\mathrm{OC}^1}$; hence, if r_s be the resistance of the field coils, the extra resistance required is $\frac{V}{\mathrm{OC}^1}-r_s$. If we wish to take into account the effect of armature reaction we proceed as in Fig. 62, where OD is drawn, as in the case of shunt generators, to show the weakening of the field by armature reaction, and is obtained from a short circuit test. Thus, suppose $\mathrm{OC}^1=5$ ampères, and $\mathrm{OC}=6$ ampères, and the weakening effect of armature reaction for the given armature current was equivalent to 0·5 ampères, it means that, to obtain the speed n^1, we must have a field current of 5·5 ampères. Then, if the P.D. be 220 volts, the shunt resistance must be 40 ohms, whilst the normal shunt resistance is 36·7 ohms, so that we must insert an extra 3·3 ohms.

§ 90. **Example.** A shunt machine has the following saturation curve at 1000 R.P.M.: $I_s=2$ A., $E=200$ V.; $I_s=2{\cdot}5$ A., $E=210$ V.; straight between. The field current required to compensate for armature reaction is proportional to the armature current and is 0·25 A. when 20 A. are flowing in the armature. Find the speed as a motor when the applied P.D. is 220 V., and the armature current is 50 A., the armature resistance being 0·2 ohm., and the shunt resistance 75 ohms. (Inter-Coll. Exam., Cambridge, 1912.)

The field current $=\frac{220}{75}=2{\cdot}94$ A.

Of this, $0{\cdot}25\times\frac{50}{20}=0{\cdot}625$ A. are required to overcome armature reaction.

Hence the nett effective value of the field current is $2{\cdot}94-0{\cdot}625=2{\cdot}315$ A.

At 1000 R.P.M. this will give

$$E=200+\frac{0{\cdot}315}{0{\cdot}5}\times 10,$$
$$=206{\cdot}3 \text{ volts.}$$

But E must $=220-50\times 0{\cdot}2,$
$=210$ volts.

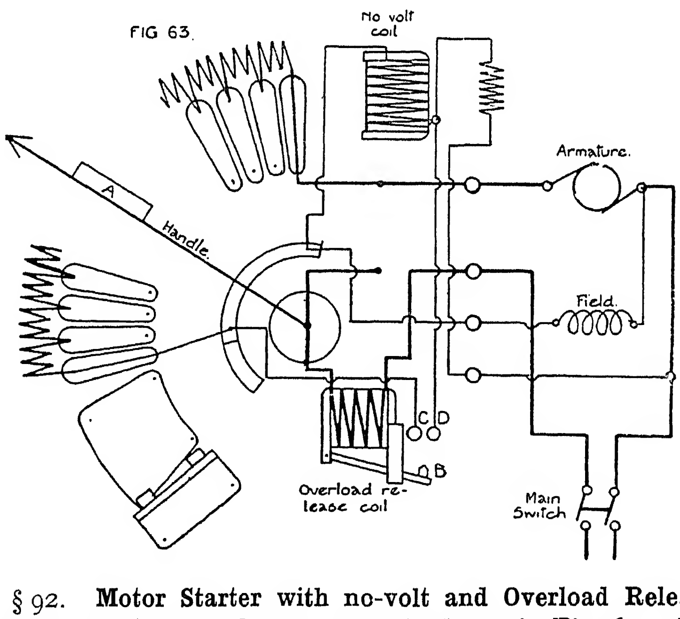
$\therefore$ the speed will be $1000\times\frac{210}{206{\cdot}3}$
$=1020$ R.P.M.

STARTERS AND REGULATORS IN PRACTICE.

§ 91. We have seen in §§ 84-86, the function of starters and regulators for motors and indicated a method of calculating the necessary resistances. We shall now devote a small space to describing how the various requirements are satisfied in practice.

A motor starter is usually provided with two special safety devices: A no-volt release, which automatically brings the starting lever to the "off" position directly there is any failure in the current supply, and an overload release which performs the same operation in the event of a heavy overload. The fuses in the main circuit do not form a sufficiently reliable safeguard, for they take an appreciable time to melt, and in this time much harm could be done to the motor. A moderate overload current will not be sufficient to open the circuit by means of the overload release on the starter, but in a short time will melt the fuse, whereas a heavy current will open the switch instantly, before the fuse would have time to melt.

FIG 63.



§ 92. **Motor Starter with no-volt and Overload Release.** A good example of modern practice is shown in Fig. 63. The no-volt coil is connected across the mains (sometimes it is

connected in series with the shunt winding of the motor), and its operation is as follows: In the starting, or "off," position, the handle is down. To start the motor the main switch is closed, and this excites the no-volt coil. The lever is then gradually moved to the right into the running position, when the no-volt coil, which is a small electro-magnet, holds on to it by attracting the small iron armature A. Should the current, for any reason, fail, the coil becomes deënergized, and the lever is returned to the starting position by the action of a spring, thus automatically preventing the motor from being started again without the full starting resistance being in the circuit.

The protection against a heavy overload current is provided by the electro-magnet shown in the lower part of the diagram. When the current exceeds a certain value, the iron armature B is drawn upwards, and thus contact is made between the points C and D. This short-circuits the no-volt coil, and the circuit is opened as explained above, owing to the magnet of the no-volt being deënergized.

§ 93. **Starter Regulators.** The starters described above permit of no regulation of the speed of the motor by resistance in the armature circuit. This drawback is overcome by the arrangement to be described, which is illustrated in Fig. 64. The action of the no-volt coil and overload coil is the same as before, and need not be described again. It will be seen that there are two arms A and B, which are connected electrically by the flexible lead C, while B is controlled by a spring and magnet as before, A being mechanically free. To start the motor. A is moved from left to right until the auxiliary lever B holds on to the magnet D, and then the motor is regulated as desired by moving A so as to cut out or insert resistance. On failure of supply or excess current, the auxiliary lever B is released and breaks the circuit.

§ 94. **Shunt Field Regulators.** An excellent example of a shunt regulator is shown in Fig. 65. The principal function of the apparatus is to regulate the strength of the field by regulating the resistance in series with it, which is done by moving the arm P to the right or left, as the case may be. But the regulator under consideration has a special feature, which

makes it impossible to start the motor except with full excitation, which is necessary lest the speed of the motor be too great.

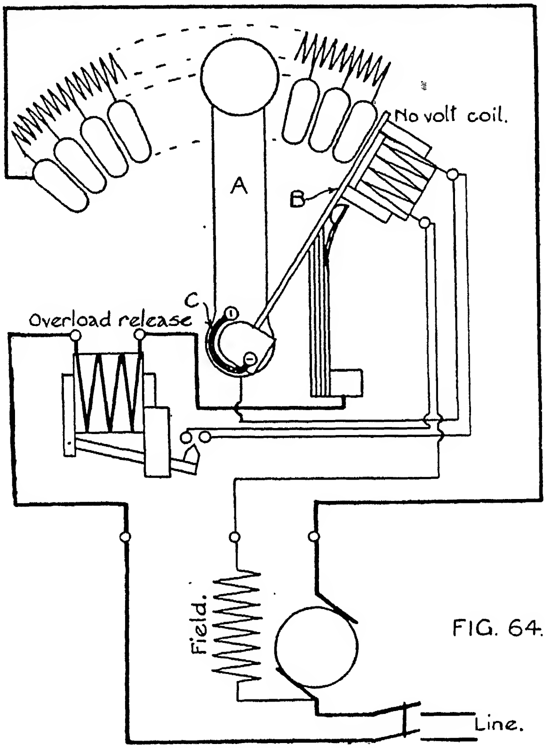


FIG. 64.

This is effected by an interlocking arrangement, consisting of a simple coil with a hinged armature, which carries a small short-

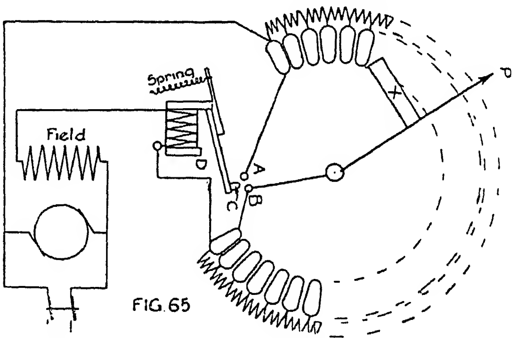


FIG. 65

circuiting tip C. This tip C is held against the posts A and B by means of a spring, so that the whole of the resistance is

short-circuited, no matter in what position the arm of the regulator may be, and full field strength prevails in the motor. In order to be able to raise the speed of the motor by inserting resistance in series with the field, it is necessary first to remove the short-circuit between A and B, which is done by moving the handle of the regulator to the left until the fibre block X pushes the armature of the coil away from A and B. When C is brought into magnetic contact with D, the excitation of the coil will be sufficient to hold it there, and it will hold on so long as any current flows in the field, thus leaving the shunt regulator unhampered in any way.

On failure of supply, or when the motor is switched off, the excitation of the coil ceases, and the spring immediately pulls the armature away from D into its position of short-circuiting A and B, and thus full field conditions are established ready for the next time the motor is started.

§ 95. **Series and Shunt Combined Starter Regulator.** An arrangement which combines the features of the series starter-regulator and a shunt regulator is shown in Fig. 66. The left-hand lever, A, which alone has a handle, is free, while the lever B, which makes the main circuit, is controlled by a spring, and the two are connected electrically by a flexible wire. To start the motor the arm A is moved from left to right until B is held by the no-volt coil, full resistance being then in the armature circuit, and none in the field, so that full field strength obtains in starting, as it should. When the motor has been started in this way, its speed can be regulated as desired; with the arm C covering segment A there will be no resistance in either the armature or field circuit, and the speed can be

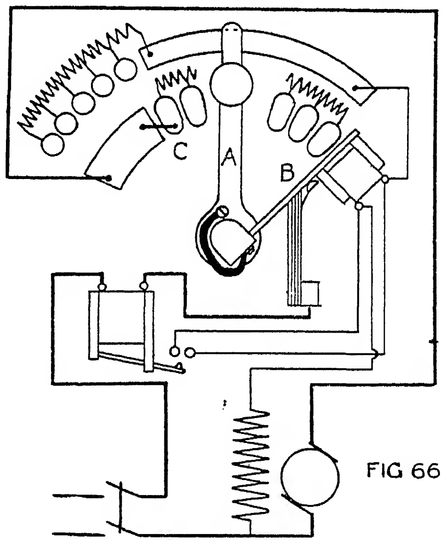
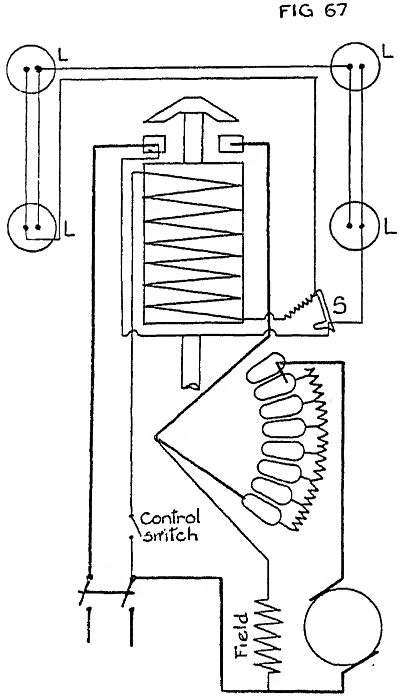


FIG 66

increased by moving the lever to the left so as to insert resistance into the field circuit and thereby weakening the field, or decreased by moving the lever to the right so as to increase the resistance of the armature circuit. The no-volt coil and overload release operate in the same way as before, from any step, during starting or while running.

§ 96. **Automatic Starters.** In many cases it is preferable that the starting of the motor should be performed by electrical operation instead of by hand, or it may be desired to operate the motor from a distance, or that the motor should be started and stopped automatically, e.g. by a float in a tank. In such cases the starter is operated by a solenoid excited by an auxiliary circuit. One of the best and most recent of these starters is shown in Fig. 67.

FIG 67

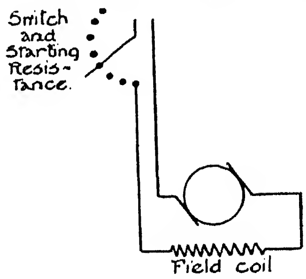


In the "off" position the upper plunger of the solenoid is kept up by a spring, and the lower plunger is down at the bottom, so that all the resistance is in the motor circuit. As soon as the control switch is closed the upper plunger, to which the main switch is attached, is instantaneously pulled down, and the main circuit is thus completed with all resistance in. At the same time the lower plunger is drawn slowly up, being retarded by the dash-pot, and the starting resistance is cut out step by step. When the lever gets to the top of the stroke it opens the switch S, thereby putting the four lamps L into the solenoid circuit to reduce the operating current to just the value necessary to hold in the plungers. These lamps are wired two in parallel and two in series so that the failure of one lamp will not seriously disturb the circuit.

On failure of supply, or when the control switch is opened, the solenoid becomes deenergized, the main switch is immediately opened by the spring, while the lower lever falls and inserts the resistance in the armature circuit ready for the next start.

SERIES MOTORS.

§ 97. In a series motor the field winding is connected in series with the armature and a controlling resistance, so that a single current flows through all of them. The connections are shown diagrammatically in Fig. 68. The chief application of series motors is to traction, for which purpose they are largely used in this country. The particular qualities which suit them for this work will be seen presently.



Series Motor.

FIG. 68.

§ 98. **Torque Current Relations.** Let I be the current taken by the motor, then, according to equation (33) above we can write:

$$M \propto \Phi I.$$

At low flux densities the flux Φ is approximately proportional to the current and we have:

$$M \propto I^2, \text{ nearly.}$$

As saturation is approached the rate of increase of the flux with the current falls off rapidly, and, when the iron is saturated it becomes more nearly true to say that $M \propto I$. We see then that, at first, the torque increases very rapidly with the current, but that the rate of increase diminishes as the current increases and the iron becomes saturated.

§ 99. **Speed Current Relations.** If E be the back E.M.F. of the motor we can write

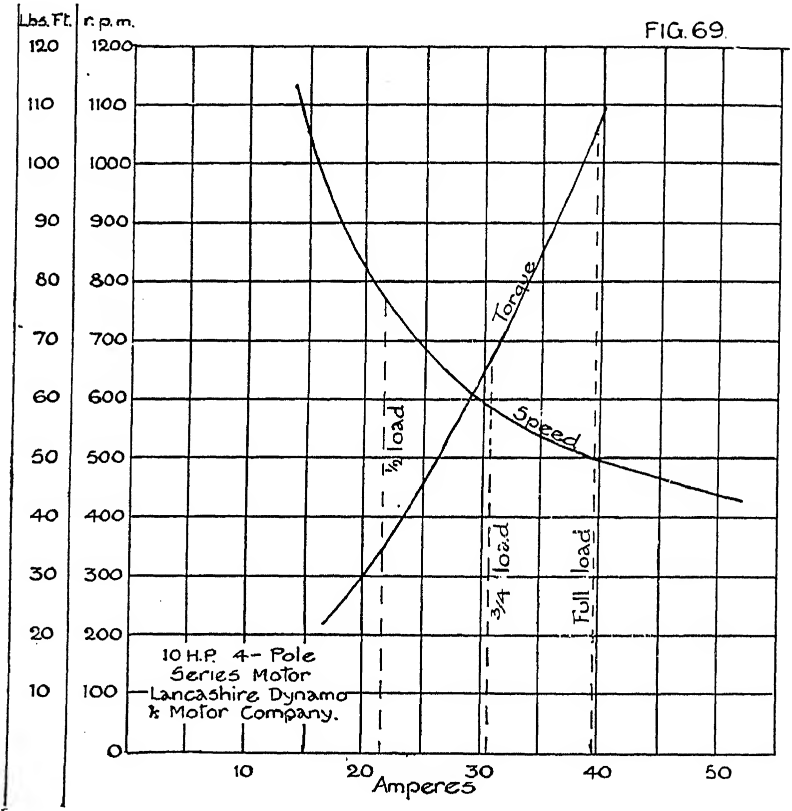
$$E \propto \Phi n;$$

and, therefore, if R be the whole resistance of the motor,

$$n \propto \frac{E}{\Phi} \propto \frac{V - IR}{\Phi};$$

and, at first, Φ is proportional to I. Thus at low values of the induction, the curve connecting the speed and the current is approximately a rectangular hyperbola, but it becomes more nearly a straight line when the iron is saturated. In other words, the speed decreases rapidly at first as the current increases, but the rate of decrease falls off rapidly as the current is increased. An actual example of the variation of torque and speed with current is shown in Fig. 69.

FIG. 69.



§ 100. **Speed Torque Relations.** Before the iron approaches saturation we can write

$$an = \frac{V - IR}{I}$$

and
$$M = bI^2.$$

approximately, where a and b are constants. These relations give

$$M=\frac{bV^2}{(an+R)^2},$$

which shows that the torque decreases very rapidly as the speed increases, the torque being very large when the speed is very small. As the current gets larger and the iron approaches saturation it becomes more nearly true to write

$$cn=V-IR$$

and

$$M=dI,$$

where c and d are constants. We then have

$$M=\frac{d}{R}(V-cn),$$

showing that the rate of decrease of torque as the speed increases is now nearly constant.

The torque-speed curve for the motor of Fig. 69 is shown in Fig. 70.

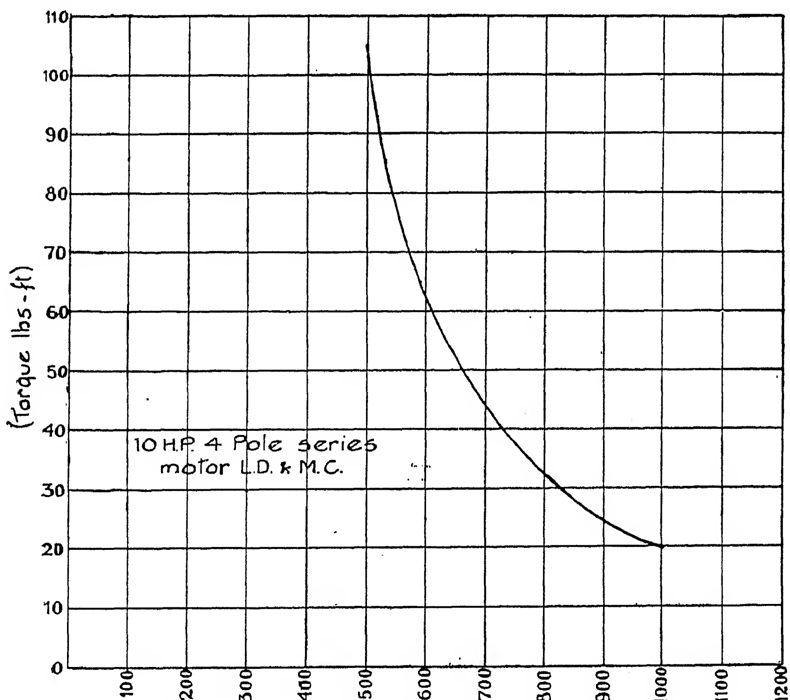


FIG 70.

It is the relation between torque and speed which makes the series motor so well suited to traction work. At starting, the torque demanded from a railway motor is much larger than that required to maintain a steady speed. Thus the traction motor is required to give a large torque when the speed is small, and to be such that the speed rises as the torque demanded decreases. These conditions are fulfilled by the series motor.

It will be seen from the above equations that the result of inserting resistance in the motor circuit is to diminish the speed for a given value of the current or torque.

It should be noted that, owing to the rapid increase of speed with decreasing load, a series motor should never be run unloaded, or it will run faster and faster until it flies to pieces; for this reason they should be geared to their load and not connected by belts, for in the event of a mishap to the latter the motor would probably come to grief.

§ 101. **Control of Series Motors.** A series motor must always have some resistance inserted in its circuit at starting, otherwise, there being no back E.M.F., the current would be excessive and burn out the motor. As the speed rises this resistance is gradually cut out until full speed is reached. The same resistance can be used for regulating the speed, but this is open to the objection of bulkiness and the large waste of power involved below top speed.

§ 102. **Starting Resistances for Series Motors.** The values of the resistances in the several steps can be approximately calculated as in the case of shunt motors. Thus, if there be n resistances in the starter, $r_1\ r_2 \ldots r_n$, and R be the total resistance of the motor, I_1 denoting the maximum current, and I_2 the minimum, we have in succession,

$$I_1 = \frac{V}{(r_1 + r_2 + \ldots\ldots + r_n) + R}$$

$$\left\{ \begin{aligned} I_2 &= \frac{V - E_1}{(r_1 + r_2 + \ldots\ldots + r_n) + R} \\ I_1 &= \frac{V - E_1}{(r_2 + r_3 + \ldots\ldots + r_n) + R} \end{aligned} \right.$$

$$\begin{cases} I_2 = \dfrac{V - E_2}{(r_2 + r_3 + + \ldots\ldots r_n) + R} \\ I_1 = \dfrac{V - E_2}{(r_3 + r_4 + \ldots\ldots + r_n) + R} \end{cases}$$

etc., etc., which lead to the expression for n as before, viz.

$$n = \frac{\log \dfrac{V}{I_1 R}}{\log \dfrac{I_1}{I_2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (40).$$

To find the several resistances we have:

$$r = \frac{V}{I_1} - R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41),$$

$$r_1 = (R + r)\left(1 - \frac{I_2}{I_1}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (42),$$

$$r_n = r_1\left(\frac{I_2}{I_1}\right)^{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (43),$$

where $r = r_1 + r_2 + \ldots + r_n$ = the total starting resistance.

To find the speeds corresponding with the steps 1, 2, 3, etc., we must also have the saturation curve of the motor.

§ 103. **Example 1.** For a two-pole series motor I denotes the current in amps., and Φ denotes the *total* armature flux. The connection between I and Φ, experimentally obtained, was as follows:

Values of I	0	10	20	30	40	50	60	70	80	90	100
Values of $\Phi \cdot 10^{-6}$	0	7·2	10·7	12·2	12·7	12·8	12·9	12·8	12·7	12·5	12·3

The resistance of the machine is five ohms, and there are 180 conductors on the armature. Show by means of curves, drawn to scale, the connection between torque and current, and between torque and speed, when the machine is supplied with current at a constant P.D. of 500 volts. (Mech. Sc. Trip. Part I. 1906.)

We have to find the torque and the speed. We have:

$$M = \text{Torque} = 0{\cdot}117 p\Phi z . 10^{-8} I = 0{\cdot}117\Phi . 180 . 10^{-8} . I,$$
$$= 0{\cdot}21\Phi I . 10^{-6} \quad \ldots\ldots\ldots \quad \text{(i)}.$$

The speed depends on the back E.M.F. and flux:—

$$E = \Phi . 180 . n . 10^{-8}.$$

$$\therefore \qquad n = \frac{10^6 E}{1 \cdot 8 \Phi} \text{ revs. per. sec.} = \frac{10^8 E}{3\Phi} \text{ R.P.M.} \quad \ldots\ldots \text{(ii)},$$

$$\text{and} \qquad E = V - Ir = 500 - 5I \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}.$$

From (i), (ii) and (iii) we can draw up the following table: Calculate E from (iii), and so find the speed from (ii). The torque is calculated from (i). The results have been plotted in Fig. 71.

I	0	10	20	30	40	50	60	70	80	90	100
$10^{-6} \times \Phi$	0	7·2	10·7	12·2	12·7	12·8	12·9	12·8	12·7	12·5	12·3
E	500	450	400	350	300	250	200	150	100	50	0
n	∞	2083	1245	952	784	649	516	390	263	133	0
Torque	0	15·1	44·9	76·8	1068	132	163	188	212	235	258

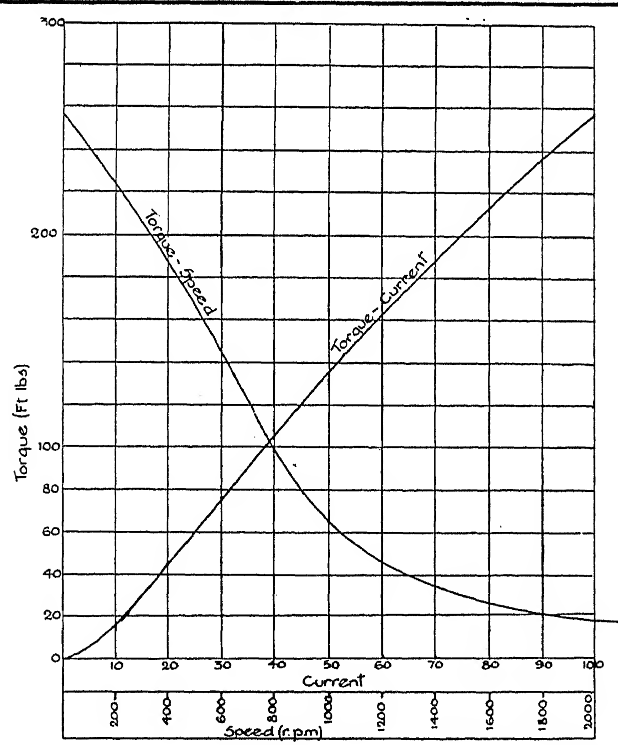


FIG. 71

2. Fig. 72 is the external characteristic of a series wound dynamo, whose resistance is 1 ohm, when run at a speed of 300 R.P.M. Plot a curve showing the relation between the torque (ft.lbs.) developed by the machine, when working as a motor, and the current, assuming that there is no core loss or friction. (Mech. Sc. Trip. 1908.)

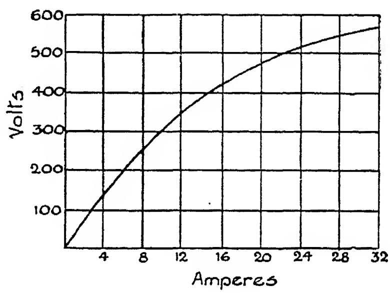


FIG. 72.

In Fig. 72 V is the terminal pressure of the dynamo; add to this I_aR and we obtain E, the total E.M.F. generated for a given value of the current.

Now $E = \psi n_0$, where ψ is the Induction Factor corresponding to any particular value of the current, and n_0 is the speed (R.P.S.) at which the machine was run, viz. 5 revs. per sec.

$$\therefore \qquad \psi = \frac{E}{n^0} = \frac{E}{5}.$$

Thus we find the Induction Factor corresponding to any given current; multiply this by $0{\cdot}117 I_a$ and so obtain the

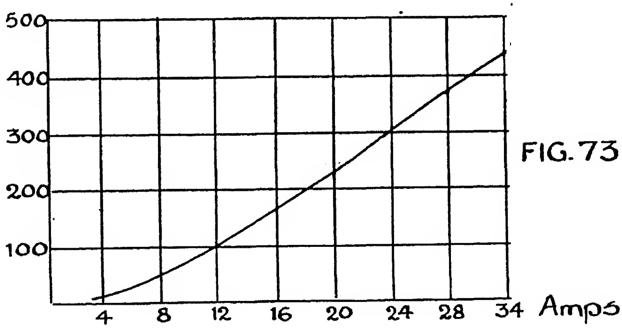


FIG. 73

torque corresponding to the same current. Proceeding in this way we have the following table (remembering $R_a = 1$).

The result is plotted in Fig. 73:

I	4	8	12	16	20	24	28	32
V	190	250	350	425	475	520	550	560
E	144	258	362	441	495	544	578	592
ψ	28·8	51·6	72·4	88·2	99·0	108·8	115·6	118·4
M ..	13·5	48	102	164	222	305	376	442

3. The following data refer to the saturation curve of an enclosed series machine for a speed of 580 R.P.M.:—

Field Current (*separately supplied*)—

50 100 150 200 250 300 350 400 amps.

Terminal P.D. (on open circuit)—

210 355 440 488 520 545 565 582 volts.

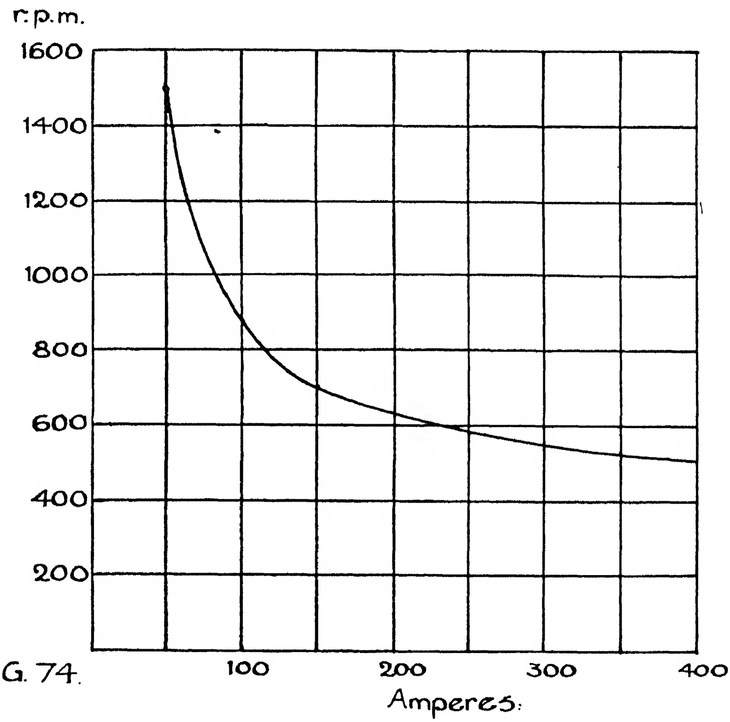


FIG. 74.

Deduce a curve connecting speed and current supplied when the machine is run as a motor off 550 volt mains. The combined resistance (hot) of armature and field is 0·11 ohm. (R.N.C., Greenwich, 1911.)

We proceed as follows: When the current is 50 ampères

the back E.M.F. $=550-50\times\cdot 11,$

$=544\cdot 5$ volts.

But, with 50 ampères in the field, the E.M.F. induced at 580 R.P.M. is 210 volts; therefore, to generate an E.M.F. of 544·5 volts at this excitation, the speed must be

$$\frac{544\cdot 5}{210}\times 580=1504 \text{ R.P.M.}$$

Proceeding in this way, we find, for the values of the current given above, the following values for E and n:—

Current I ..	50	100	150	200	250	300	350	400
Back E.M.F.	544·5	539	533·5	528	523·5	517	511·5	506
R.P.M. ..	1504	880	705	628	585	550	523	505

The curve is shown in Fig. 74.

4. A series wound crane motor on a 440 volt circuit can raise, at full load, 2 tons at 25 feet per minute, the gear having an efficiency of 80%. The efficiency of the motor itself is 84% at full load, and its resistance 1·5 ohms. Sketch an assumed probable saturation curve for the motor, and estimate the speed at which it will lift loads of $1\frac{1}{2}$ and $2\frac{1}{2}$ tons respectively, on the assumption of constant overall efficiency of motor and gear. (B.Sc., London University, 1911.)

Take the curve AB (Fig. 75) as saturation curve. Now, to lift 1 ton at v ft./min. requires

$$\frac{2240v}{33{,}000}\times 746 \text{ watts.}$$

Hence the input to the motor is

$$\frac{100}{84}\times\frac{100}{80}\times\frac{2240v}{33000}\times 746 \text{ watts.}$$

$\therefore$ at 440 volts, we have, when lifting one ton,

$$I = \frac{100 \times 100 \times 2240v \times 746}{84 \times 80 \times 33000 \times 440} = 0{\cdot}172v.$$

When lifting 1·5 tons, $I = 0{\cdot}258v$.

" " 2·5 " $I = 0{\cdot}429v$.

" " 2·0 " at 25ft. /min., $I = 8{\cdot}6$ amps.

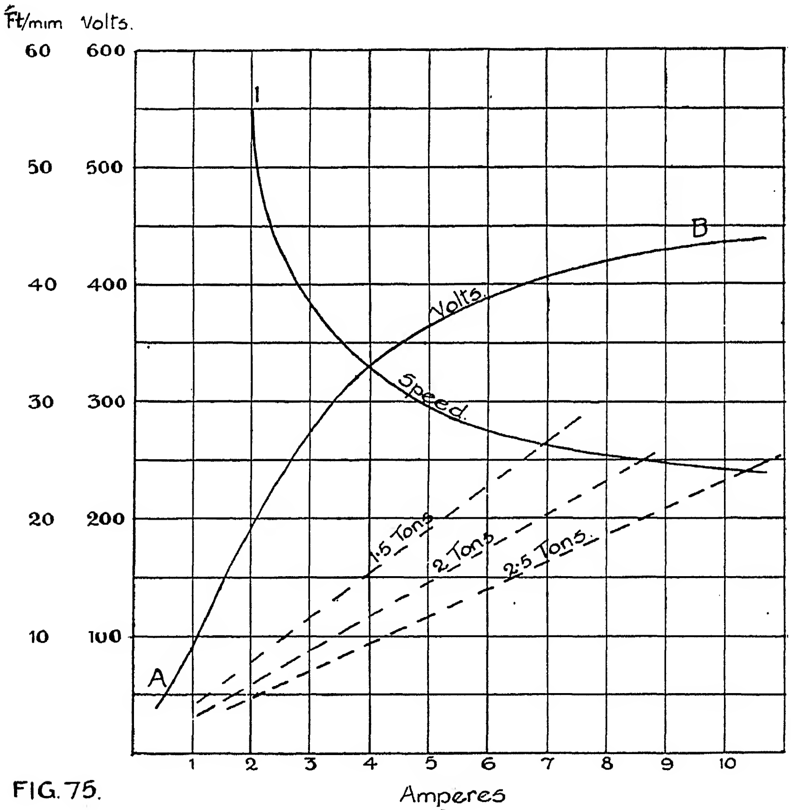


FIG. 75.

We must now draw the speed-current curve, as in the last example, and then, if we draw the lines $I = 0{\cdot}258v$. and $I = 0{\cdot}429v$., the points where they intersect the $I - v$ curve for the motor give the required speeds, 26·5 and 24 feet per minute respectively.

5. A 500 V. series motor has a speed of 600 R.P.M. when taking a current of 50 A. What will be the speed when it is

nected in series with another similar motor, with 1 ohm in ies, the current and pressure being the same as before? The istance of each motor is 0·5 ohm.

In the first case

$$E = 500 - 50 \times 0{\cdot}5 = 475 \text{ v.},$$

d

$$n = 600 \text{ R.P.M.}$$

the second case we have

$$2E = 500 - 2 \times 50.$$

$$E = 200.$$

the new speed $= \dfrac{200}{475} \times 600 = 253$ R.P.M.

COMPOUND DIRECT-CURRENT MOTORS.

§ 104. When it is necessary that a motor should be able carry heavy overloads for a short time, without a tendency race under light loads, or run at absolutely constant speed, becomes necessary to have a compound-wound motor.

§ 105. The first of the above conditions is necessary with rtain machine tools, such as punching and planing machines, here the motor is heavily overloaded upon the first contact of e tool with the metal. The extra torque is supplied by a rge rush of current into the armature, but this can be duced if the shunt-wound driving motor be provided with me series turns on its field, so wound that the latter is auto- atically strengthened by the increase of current. The ries turns must be wound the same way as the shunt turns, that they add to the shunt field, and this is called *cumulative mpounding*. What happens is this: There is a sudden demand r more torque, and the armature current increases at once; is current passes round the series coils of the field magnets, id so strengthens the field at the same time. Hence the xtra current will not have to be so large as with the un- rengthened field, since the torque is proportional to the roduct of armature current and flux.

§ 106. When we wish to increase the speed of a shunt otor we have to weaken the field. Now as the load is in- reased the speed of a shunt motor falls off slightly; and there- re, in order to maintain constant speed, we must provide a eries field as well, but in this case it must be wound so as to

oppose the shunt field. This is called *Differential Compounding*. When the load increases, the armature current increases, and this, passing through the series turns, weakens the field and so increases the speed.

A study of the example in § 107 will show that, so long as we are concerned with E.M.F.'s corresponding with the straight portion of the O.C. characteristic, it is possible, with a fixed number of series turns, to provide that the speed shall be constant for all loads.

§ 107. **Example.** A shunt motor is tested as a dynamo on open circuit with the field separately excited, in order to obtain the saturation curve. It is found that the first part of the curve is a straight line through the origin and such that 0·3 A. in the field gives an E.M.F. of 35 V., and the upper part of the curve is shown in Fig. 76. A short circuit test at the same speed showed that 0·51 A. are required in the field to give 80 A. in the armature. The resistance of the armature is 0·16ω., that of the shunt winding 44ω., and the number of field turns is 2000. It is required to make the motor run at the same speed with 80 A. as when unloaded, the P.D. of supply being 220 V. Find the number of series field turns required, assuming that their resistance will be 0·09ω. Also find the fluctuation of speed for values of the armature current between 0 and 80 A.

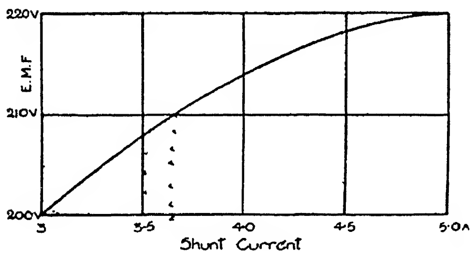


FIG 76

The constant value of the shunt current $= \frac{220}{44} = 5$ A.

∴ the shunt ampère-turns = 10,000.

With $I_a = 80$, the E.M.F. in the short circuit test must have been $80 \times 0{\cdot}16 = 12{\cdot}8$ V., and the open circuit test shows that this requires a shunt current equal to $\frac{12{\cdot}8}{35} \times 0{\cdot}3 = 0{\cdot}11$ A., when there is no armature reaction. The short circuit test

›ws that, with armature reaction, a shunt current of 0·51 A. ·equired for the same E.M.F. Therefore, the demagnetizing ɜct of an armature current of 80 A. is represented by (0·51)·11) × 2000 = 800 ampère-turns, and in proportion for other nature currents.

When the series turns are included, the internal drop with A. will be 80 (·09 + ·16) = 20 V.; therefore the generated ɪ.F. must be 220 − 20 = 200 V. At the speed of test this ụires 6000 effective ampère-turns on the field. The actual ıpère-turns are 10,000, of which 800 are neutralized by nature reaction. Therefore there remain 10,000 − 800 − ɔo = 3200 ampère-turns to be neutralized by the series :ns. Therefore the number of turns required $= \frac{3200}{80} = 40$.

To find the speed fluctuation: The calculations should understood from the following table without further ex- ınation:

ɪrmature current ..	0	20	40	60	80
›hunt ampère-turns neutralized by armature reaction	0	200	400	600	800
Shunt ampère-turns neutralized by series turns	0	800	1,600	2,400	3,200
Total shunt ampère-turns	10,000	10,000	10,000	10,000	10,000
∴ effective field ampère-turns = ...	10,000	9,000	8,000	7,000	6,000
Generated E.M.F. (from Fig. 76) at test speed ..	220	218·3	214	208	200
$V - I_a \times 0{\cdot}25$ (actual back E.M.F.)	220	215	210	205	200
∴ ratio of speed to speed of test = ...	1	0·985	0·982	0·985	1

EXAMPLES.

1. A motor has four poles, and runs at 600 R.P.M. with 200 volts P.D.; the armature is wave-wound. Find the gross theoretical torque for an armature current of 50 A., neglecting the ohmic drop in the armature. (Inter-Coll. Exam., Cambridge, 1912.)

2. A 250-volt shunt motor with an armature resistance of 0·1 ohm, and a field resistance of 600 ohms, gives 10 H.P. at 85% efficiency. Calculate the back E.M.F., and the armature current. (Mech. Sc. Trip., Cambridge, 1912.)

3. In a two-pole shunt motor the flux per pole is $1{\cdot}2 \times 10^6$ lines, the number of armature wires is 534, the armature resistance is 0·1 ohm, the speed is 1000 R.P.M. Find the gross torque if the P.D. is 110 volts. (Mech. Sc. Trip., Cambridge, 1912.)

4. A shunt motor runs light with $I_a=2$ at 600 R.P.M. and $V=250$. The armature resistance is 0·5 ohm. If the armature reaction is such that the reduction of flux is proportional to the armature current, and is 2% when the armature current is 10 A., find the speed when the load on the shaft is 10 H.P. (Inter-Coll. Exam., Cambridge, 1911.)

5. If the saturation curve of the machine in Ex. 17, p. 74 were obtained at 800 R.P.M. Find the speed as a motor when $I_a=85$ A.

6. It is required to design a starter for a certain shunt motor. Show that, if the copper loss in the armature at full load is 4% of the input, eight contacts on the starter will suffice to restrict the current rush to 1·5 times the normal full load current. Calculate the resistances to be inserted between each pair of adjacent stops, if the resistance of the armature is 0·5 ohm. (R.N.C., Greenwich, 1912.)

7. The following particulars refer to a 35 H.P. shunt motor:

Number of poles 4

Air-gap cross section at pole-face ....450 cm^2.

Flux density 5500

Armature winding 2-circuit

Peripheral conductors 790

Total current ....140 A.

Calculate the gross theoretical torque.

If the rotational losses = 1000 watts, estimate the nett shaft torque.

The resistance of the armature = 0·15 ohm., and the applied P.D. = 220 V. Calculate the speed and the B.H.P. when taking 140 A. in the armature.

8. A series motor has a resistance of 3 ohms, and is taking a current of 20 A. at 500 V., and running at 600 R.P.M. Find the torque it is exerting. (Inter-Coll. Exam., Cambridge, 1904.)

9. A series motor has a resistance of 2 ohms, and is supplied at 200 V. The corresponding currents and speeds are given below. Deduce the corresponding torques. If two of these motors are placed in series across the mains with an extra resistance of 1 ohm, find the speed when the current is 25 A.

Current, amps.......	10	20	30	40	50
R.P.S.	20	12·6	9·7	7·6	6·0.

(Inter-Coll. Exam., Cambridge, 1906.)

10. The armature of a 4-pole wave wound motor has 396 conductors on its periphery; it is 10″ diameter, and the effective length of the iron is 12″. The pole pieces occupy 0·6 of the whole circumference, and it may be assumed that the flux is normal to the poles, and to the armature. If $\mathcal{B} = 8000$ in the air-gaps, find the torque for a current of 30 A. (Inter-Coll. Exam., Cambridge, 1913.)

11. A 200 V. series motor, whose resistance is 0·75 ohm, runs at the following speeds with the given currents:

Current, amps.......	20	30	40
R.P.M.	900	700	600

Find the speed when taking 25 A. with 4 ohms in series with the motor.

12. In a two-pole shunt motor the flux per pole is $1{\cdot}2 \, . \, 10^6$ lines, the number of armature conductors is 534, the resistance

of the armature is 0·1 ohm, the speed is 1000 R.P.M. Find the gross torque if the P.D. be 110 V.

13. A certain series motor runs at 800 R.P.M. when the P.D. is 400 V., and the current 100 A. If the P.D. be reduced to 300 V. while the torque remains constant, find the new speed. Armature resistance = 0·1 ohm. (R.N.C., Greenwich, 1913.)

14. A six-pole motor has a wave-wound armature with 250 conductors. The resistance of the winding is 0·0165 ohm, and the total brush contact drop is 1·6 V. At P.D. of 110 volts the speed is 240 R.P.M., and the armature current is 365 A. Find (i) the total torque exerted, (ii) the flux per pole. (R.N.C., Greenwich, 1913.)

15. A starter is required by means of which the motor referred to in Ex. 3, p. 102, can be thrown on the line without the starting current ever rising above 440 A. or sinking below 275 A. Find the number of steps required and the resistance between each step.

CHAPTER IV.

EFFICIENCY AND LOSSES IN D.-C. MACHINES.

§108. **Sources of Loss in D.-C. Machines.** We must now consider the various sources of loss in direct-current dynamos and motors; that is to say, how it is we do not obtain in one form as much energy per second as we put into the machine in another form, and what becomes of the energy which has apparently disappeared.

We may divide the total loss of power into two classes—"rotational losses" and "copper losses."

The rotational losses consist of those due to friction, hysteresis, and eddy currents, and are so called because they arise purely from the armature rotation, whilst the copper losses occur in the shunt coils, in the armature and commutator, and in the series field, and arise purely from the flow of currents in the windings.

§ 109. **Hysteresis Loss.** When a piece of iron is taken round a magnetic cycle, a certain expenditure of energy is incurred on account of hysteresis, which loss is, by Steinmetz' empirical law, approximately proportional to $\mathscr{B}^{1 \cdot 6}$ for a given iron, and for medium values of the maximum induction. Now in a generator or motor, the armature iron is continually going through magnetic cycles, so that there is an hysteresis loss, which is proportional to $n\mathscr{B}^{1 \cdot 6}$, where n is the speed of rotation, and $\mathscr{B}$ the maximum induction or flux-density. Thus the hysteresis loss is given by P_H = loss in watts per unit volume $=\eta n \mathscr{B}^{1 \cdot 6} . 10^{-7}$, where η is a constant. If $\mathscr{B}$ be measured in lines per square centimetre, n in cycles per second, then P_H is

given in watts per cubic centimetre of iron and $\eta = 0{\cdot}00194$ for first-class iron to $0{\cdot}0038$ for inferior material.

If $\mathcal{B}$ be measured in lines per square inch, P_H is given in watts per cubic inch, and $\eta = 0{\cdot}0016$ to $0{\cdot}0032$.

§ 110. **Eddy Current Losses.** Owing to the changing flux in the armature, eddy currents are induced in the armature iron and power is consumed thereby, this power being proportional to the square of the speed, the square of the induction, and the square of the thickness of the laminations (see p. 14). Thus the loss can be expressed approximately in the form $P_E =$ loss in watts per unit volume $= \xi n^2 \mathcal{B}^2 d^2 . 10^{-12}$ where ξ is a constant.

If we measure $\mathcal{B}$ in lines per square centimetre, and d, the thickness of the laminations, in centimetres, P_E is given in watts per cubic centimetre if $\xi = 59{\cdot}1$ for an average iron; whilst if $\mathcal{B}$ be given in lines per sq. in., and d in inches, P_E is given in watts per cubic inch if $\xi = 150$.

Hence, combining the above formulae, the total iron loss is expressed approximately by

$$P_i = \text{loss in watts per unit volume,}$$
$$= \eta n \mathcal{B}^{1{\cdot}6} . 10^{-7} + \xi n^2 \mathcal{B}^2 d^2 . 10^{-12} \quad \ldots\ldots\ldots (44).$$

In practical designing it is important to be able to predetermine the iron loss, and it is found that these formulae are not, on the whole, satisfactory. They generally lead to results which are too small, chiefly on account of the non-uniformity of the maximum induction in different parts of the armature, and experiment leads to the following formula—

Total iron loss in watts per pound

$$= \text{constant} \times \text{cycles per second} \times \text{millions of lines per sq. in.}$$

and Cramp* gives $1{\cdot}7$ to $1{\cdot}9$ as the value of the constant, if the density be taken as the average value below the teeth, and the discs are not more than $0{\cdot}02''$ thick.

* *Continuous Current Machine Design*

Sometimes, however, the theoretical formulae will give a fair approximation to the truth, especially if the lamination of the armature be good; and, for dividing the experimentally determined total iron loss, of a given machine, into the components due to hysteresis and eddy currents respectively, we must take the former as proportional to speed $\times \mathcal{B}^{1.6}$ and the latter as proportional to $(\text{speed})^2 \times \mathcal{B}^2$. When using any well-known brand of iron it is usually possible to obtain curves which give the loss in watts per pound for various values of $\mathcal{B}$, at a given frequency, such as given in Fig. 77.

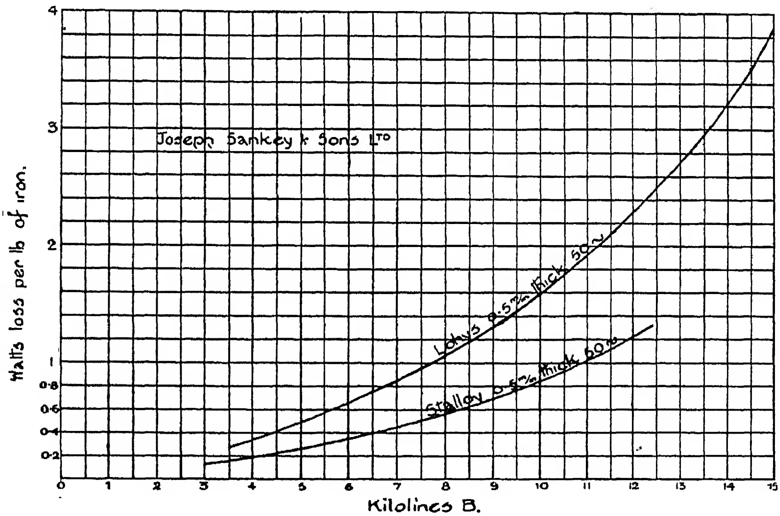


FIG. 77.

The iron loss may be somewhat reduced by using one of the special alloy irons now on the market, such as "stalloy" iron, which has a high permeability and low iron losses. "Stalloy" is a silicon-iron alloy containing about 3 per cent. of silicon, but it is much more expensive than the more generally used lohys iron, and for some unknown reason, the full advantage of stalloy iron is not obtained with direct-current work; so that, while it is customary to use stalloy iron for alternating current work, lohys iron is generally used for direct-current machines.

Fig. 78 shows in detail the losses of a modern four-pole machine.

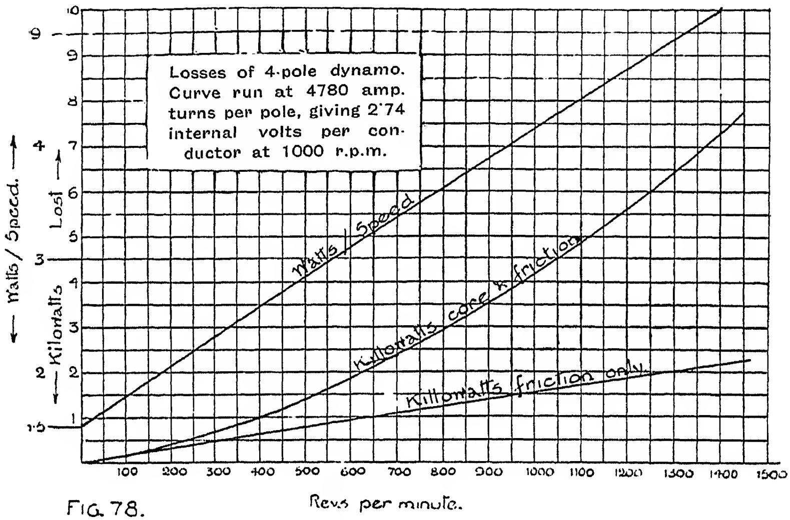


FIG. 78.

§ 111. **Friction Loss.** This includes bearing friction and brush friction, and, generally speaking, it is taken to include the power lost in air friction and in churning up the air, the whole being taken as proportional to the speed, although the "windage" is certainly not exactly proportional to the speed. The bearing friction and windage will vary from about 2 per cent. of the output for small machines to 0·5 per cent. for very large ones, the brush friction being about one-half this.

§ 112. **Constancy of Rotational Loss in Shunt and Compound Machines.** It will be seen, then, that for a given armature, the rotational loss will be constant if the speed and air gap induction are constant. Moreover, small errors in the estimated losses have a negligible effect on efficiency, as they are of the second order of small quantities. Now the main M.M.F. round the magnetic circuit of a shunt or compound machine is normally supplied by the shunt turns and the iron is saturated. Hence only a large change in the shunt current

can have sufficient effect on the induction to have a serious influence on the losses. But the P.D. on the shunt circuit of a shunt or compound generator is normally constant, and so the variation of induction will not, in general, be large enough to matter from the point of view of losses. Further, constant speed is the normal working condition for a generator, and is a characteristic property of shunt and compound motors. Thus, *in estimating the efficiency of a shunt or compound motor or generator of normal type we may take the rotation losses as constant for all loads.*

§ 113. **Copper Loss.** This occurs in the field winding, the armature coils, the commutator segments and the brushes. In shunt machines the field copper loss is practically constant, but with compound wound machines there is in addition a variable copper loss in the series field coils. All these losses can be fairly accurately predetermined for a given machine when we know the currents carried in the different circuits, and the resistance of the circuits.

To calculate the copper loss in the armature we have:

The resistance from brush to brush

$=\rho\times$length of one path of the winding from brush to brush

$\div$the cross-section of all parallel paths.

In addition there is the loss involved in the reversal of current during commutation. The energy stored in a coil of self-inductance L (§ 10) and carrying a current I is $\frac{1}{2}LI^2$. During reversal this energy has to be dissipated and supplied again, and the frequency of reversal is proportional to the speed. Hence the power wasted in commutation is proportional to I^2n. In shunt or compound machines, where the speed is substantially constant, this loss can be conveniently included in the copper loss group. In any case the loss is usually small, and it is a fair approximation to include it in the loss ($I_a{}^2R_a$) in the armature if R_a include the resistance of the brushes and brush contacts.

Example. Calculate the resistance of the following armature:

Number of conductors	278
,, ,, parallel paths	 2
Mean length of single turn	........136 cm.
Cross-section of conductor	........0·482 cm².
Specific resistance, cm. units	......$2 \cdot 10^{-6}$ ohm.

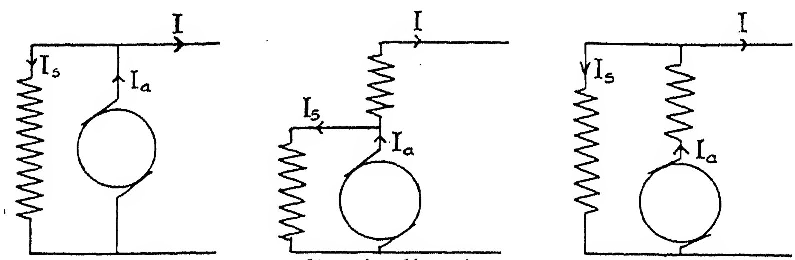
The number of conductors in series $=139$,

$\therefore$ the number of turns in series $=\frac{139}{2}=69{\cdot}5$,

$\therefore$ total length of winding from brush to brush

$$=69{\cdot}5 \times 136 \text{ cms.},$$

Hence $$R=\frac{2.10^{-6}\times 69.5\times 136}{2\times{\cdot}482}=0{\cdot}0196 \text{ ohm}.$$



Shunt Dynamo.

Short Shunt. Long Shunt.

Compound Dynamo Type

FIG. 79.

§ 114. **Efficiency of Generators.** The total lost power in any dynamo is

$$P=P_o+I_a^2R_a+I_s^2R_s+I_f^2R_f$$

where P_o=sum of hysteresis, eddy current, and friction losses, which are assumed constant (see § 112) for a generator.

$I_a^2R_a$=armature copper loss.

$I_s^2R_s$=shunt field copper loss.

$I_f^2R_f$=series field copper loss (if any).

In a shunt dynamo, $I_a = I + I_s$, and $I_f = 0$.

In a compound dynamo, if it have a *short shunt*, i.e. if the shunt be connected across the brushes, $I_a = I + I_s$, and $I_f = I_a - I_s = I$, the external current.

In a compound dynamo with *long shunt*, i.e. with the shunt connected across the terminals, we have $I_a = I + I_s = I_f$.

In all cases the output of the machine is VI, where V is the terminal P.D., and since the power taken must equal the sum of the output and the total losses, we have

$$\eta = \text{efficiency} = \frac{VI}{VI + P_o + I_a^2 R_a + I_s^2 R_s + I_f^2 R_f}.$$

§ 115. **Condition for Maximum Efficiency.** Consider first the compound machine with short shunt, so that $I_f = I$, and $I_a = I + I_s$. Then

$$\eta = \frac{VI}{VI + P_o + I_a^2 R_a + I_s^2 R_s + I_f^2 R_f},$$

$$= \frac{VI}{VI + P_o + (I + I_s)^2 R_a + I_s^2 R_s + I^2 R_f},$$

$$= \frac{V}{V + \frac{P_o}{I} + \frac{(I + I_s)^2 R_a}{I} + \frac{I_s^2 R_s}{I} + IR_f}.$$

Now since V is supposed constant the efficiency will be a maximum when the denominator is a minimum, and to find the current for which this occurs we must differentiate the denominator with regard to I, since V, P_o, I_s are approximately constant, and we shall neglect $I_s^2 R_a$, which is very small. This gives

$$-\frac{P_o}{I^2} + R_a - \frac{I_s^2 R_s}{I^2} + R_f = 0,\text{*}$$

or $$P_o + I_s^2 R_s = I^2 (R_a + R_f).$$

* The second differential coefficient of the denominator is $\frac{2P_o}{I^3} + \frac{2 I_s^2 R_s}{I^3}$, which is positive, so that the condition found makes the denominator of the expression for η a true minimum.

The left-hand side of these represents the constant losses of the machine, and the right-hand side the variable losses, for I is very nearly equal to I_a. If we put $R_f = 0$ we have the case of a simple shunt machine, and for a compound machine with a long shunt R_f is equivalent to an increase of R_a, so that the above equation shows that any direct current generator has its maximum efficiency at that load which makes the variable losses equal to the constant losses, and the same conditions obtain for a motor.

As a matter of fact, in practice a machine is not usually designed to have its maximum efficiency at normal load, but the decision of the losses is settled from quite different practical considerations, such as temperature rise, etc.

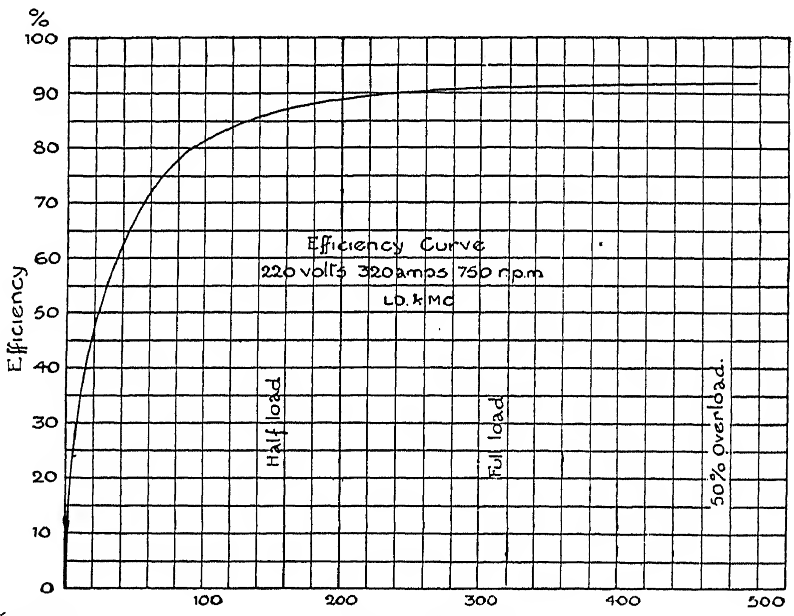


FIG. 80

In Fig. 80 is shown the efficiency curve for a modern four-pole generator designed to give 320 ampères at 220 volts, and it will be seen that the efficiency increases up to an overload of over 50 per cent., but the increase is only slight from half-load to this value, being 87 per cent. when the current is 160 ampères, and 92 per cent. at 480 amperes. Below about half-load, however, the efficiency falls off rapidly.

§ 116. **Temperature Rise.*** The power lost in electrical machinery generally reappears in the form of heat, thereby causing a rise of the temperature of the apparatus, and in dynamos and motors this temperature rise has a considerable influence on design, in fact it is one of the most important factors nowadays since the use of interpoles has rendered sparkless commutation, which used to be the chief limiting factor, possible at practically any load.

The chief source of heat in the field magnets is the copper loss in the winding; whilst in the armature, copper loss and iron loss are the causes of the temperature rise. This heat is carried away partly by conduction and convection, but mostly by radiation, and the rate at which the heat can be carried away, and therefore the rate of rise of temperature, depends on the area of the radiating surface, the difference between the temperature of the body and the temperature of surroundings, and the nature of the surface. The rise of temperature follows an exponential law approximately, and continues until the rate of dissipation of heat is equal to the rate of generation of heat.

§ 117. **Heating and Cooling Curves.** We shall suppose that a quantity of heat is being generated in the armature at a rate of Q calories per second, that the armature can radiate heat at the rate of a calories per sq. cm. per 1°C. difference of temperature, and we shall imagine the armature to be homogeneous, of mass m grammes and specific heat σ.

Let θ = temperature rise at end of time t. Then in time dt, the heat generated is Qdt, the heat dissipated is $aA\theta dt$, where A is the cooling surface, whilst the heat given to the iron is $m\sigma d\theta$. Hence we have

$$m\sigma d\theta = (Q - aA\theta)dt.$$

Now let θ_1 be the final temperature reached. Then, since $d\theta$ is now zero, we have

$$Q = a.A.\theta_1.$$

$$\therefore \quad m\sigma d\theta = aA(\theta_1 - \theta)dt.$$

$$\therefore \quad \frac{d\theta}{\theta_1 - \theta} = \frac{aA}{m\sigma}dt.$$

* For full treatment see Cramp's *Continuous Current Machine Design.*

Integrating this we have

$$\log_e \frac{\theta_1}{\theta_1 - \theta} = \frac{aAt}{m\sigma}$$

or

$$\frac{\theta_1}{\theta_1 - \theta} = \epsilon^{\frac{aA}{m\sigma}t}$$

or

$$\theta = \theta_1 (1 - \epsilon^{\frac{aA}{m\sigma}t}) \quad \ldots\ldots\ldots\ldots \quad (45).$$

When the supply of heat is stopped, and the body is allowed to cool, we have, in a similar way,

$$\theta = \theta_1 \epsilon^{-\frac{aA}{m\sigma}t} \quad \ldots\ldots\ldots\ldots \quad (46),$$

where θ is now the fall of temperature.

Although an armature is by no means homogeneous, it is found that the actual curves of rise and fall of temperature found by experiment follow the above equations fairly well. According to (45) above it would take an infinite time for the machine to reach its final temperature, but it is found that the maximum temperature is generally reached within six hours.

§ 118. As regards the permissible temperature rises in coils, if the temperature of the air be not greater than 25°C., we can allow a rise of 60°C., found by measuring the increase of resistance.*

§ 119. The ability of an armature to get rid of its waste heat depends chiefly on its radiating surface, and for this reason it is usual to arrange ventilating spaces in order to obtain a good circulation of air. The amount of heat that is liberated also increases with the peripheral speed. In totally enclosed motors the radiating surface is usually taken as the total external surface of the machine, which is sometimes increased by means of ribs on the casing.

EXPERIMENTAL DETERMINATION OF LOSSES.

§ 120. It is possible to find the efficiency of a machine either by measuring directly the output and input, or by measuring one of these and the losses. The latter, however, is often the more convenient method, as it does not require a

* Engineering Standards Committee.

full-power run to measure the losses. For any machine we have

$$\text{efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{input} - \text{losses}}{\text{input}}.$$

With a generator we usually use the first expression, and for a motor the second expression.

The copper losses need not be found experimentally; it is only necessary to measure the resistances, allowing for the temperature rise unless the resistances are measured when the windings are hot, and then the copper loss can be readily calculated. We will proceed now to describe experimental methods for finding the iron and friction losses.

The following methods apply to shunt wound motors or generators.

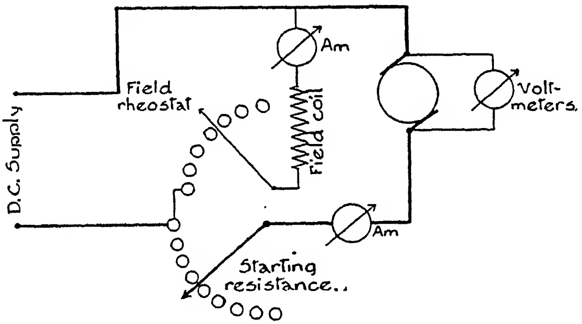


FIG. 81.

§ 121. **Rotational Losses found by running the Machine as a Motor.** Connect the machine as shown in Fig. 81, and run it at no-load and designed voltage; vary the excitation by means of the field resistance and the speed by means of a resistance in the armature circuit (not shown in Fig. 81), and so take a set of readings of the armature current and the voltmeter which is connected across the brushes, thus obtaining the total power given to the armature for any given excitation and speed. Measure the armature resistance, and calculate the armature copper loss. Then if this be subtracted from the total power given to the armature the remainder will be the iron and friction loss, which can thus be obtained for any given excitation and speed, and these are the factors upon which the loss depends (see § 112).

§ 122. **Losses found by Driving the Machine Mechanically.** The machine is connected to a small driving motor and run as a generator on open circuit, both machines being separately excited, as in Fig. 82. Then, since the generator is on open circuit, the input into motor, corrected for the motor's own losses, is the total loss of the generator due to iron and friction; for the field copper losses are separately supplied, and there is practically no armature copper loss. By driving the generator unexcited, the friction loss is obtained separately, for there is then no flux, and so no iron loss. To obtain the losses in the motor itself it is only necessary to take off the belt and measure

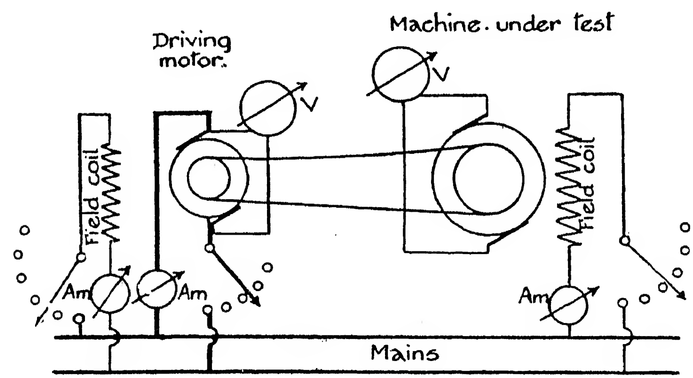


FIG. 82.

the power given to its armature, correcting for its copper loss unless this be negligible, when running at the same speed and same excitation as it had when driving the machine under test. Thus again we can find the total iron and friction loss in the machine under test.

§ 123. **Separation of Losses.** By either of the above means we can obtain a series of curves as shown in Fig. 83, giving the total loss due to friction, hysteresis, and eddy currents, for different values of the exciting current, as a function of the speed. Also, in the second case, by taking two sets of readings with the field unexcited, one with the brushes on, and one with the brushes off, we can separate brush friction.

Thus, for any speed, ON, in Fig. 83, PN is the total iron and friction (including windage) loss, PQ is the iron loss, QR is the brush friction, RN is the bearing friction and windage.

When the machine is run electrically as a motor it is not

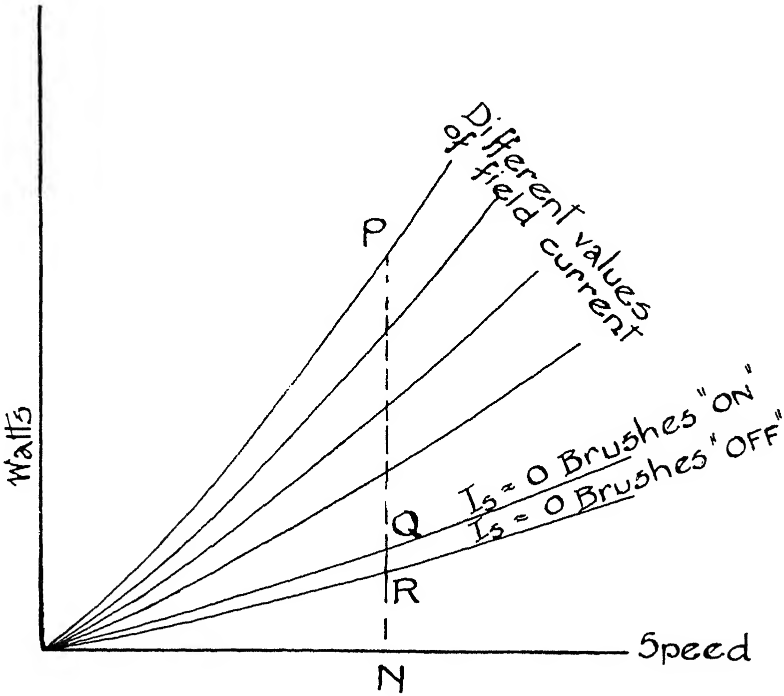


FIG. 83

possible to obtain the friction loss separately, but this can be found approximately as follows :—

For any given speed the armature voltage on open circuit is proportional to the flux, and so we can say that the hysteresis loss is proportional to $E^{1\cdot 6}$ and the eddy current loss varies as E^2 for any given speed, while the friction loss remains constant where E is the armature voltage on open circuit. Hence the total loss can be written

$$P = P_F + \alpha E^{1\cdot 6} + \beta E^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (66).$$

where P_F is the friction loss, and α and β are constants. Now the experiment enables us to draw a series of curves, for

different speeds, like those in Fig. 84, where the abscissae are the armature voltage; the machine is running as a motor so that $E=V-I_aR_a$. Then P_F is the value of P when E is zero, so we can find P_F by continuing the curves of Fig. 84 back to the P-axis by eye, or by calculation. If we take three points

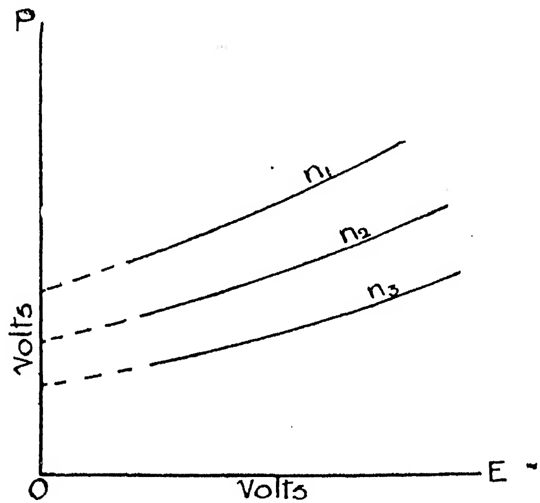


FIG. 84

on any one curve corresponding with E_1, E_2, E_3, we can obtain by substitution three equations:

$$P_F-P_1+\alpha E_1^{1\cdot 6}+\beta E_1^2=0,$$
$$P_F-P_2+\alpha E_2^{1\cdot 6}+\beta E_2^2=0,$$
$$P_F-P_3+\alpha E_3^{1\cdot 6}+\beta E_3^2=0,$$

whence α and β can be eliminated and P_F found. A fair approximation can be formed by supposing the hysteresis loss to follow the square law, when only two points on the curve are necessary, and we have

$$P_1=P_F+\alpha E_1^2,$$
$$P_2=P_F+\alpha E_2^2,$$

which lead to

$$P_F=\frac{P_1-P_2\left(\frac{E_1}{E_2}\right)^2}{1-\left(\frac{E_1}{E_2}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots (47).$$

$\therefore$ when $\mathcal{B}=6000$, and $n=1200$, the hysteresis losses

$$=\frac{300\times6^{1\cdot6}\times10^{4\cdot8}\times1200}{5^{1\cdot6}\times10^{7\cdot8}},$$

$$=360\times(1\cdot2)^{1\cdot6},$$

$$=496 \text{ watts.}$$

Thus the total iron loss at the new speed and excitation is $415+496=911$ watts.

Example 2. The following test was made on a motor provided with two pairs of brushes: The motor's field was constantly excited. With one pair of brushes and 100 volts the armature took 5·5 amps., with both pairs it took six amps. With 150 volts and both pairs it took 7·75 amps. Find the brush, hysteresis, and eddy losses at 125 volts. The armature drop can be neglected throughout.

Since the excitation is constant n will be proportional to E so we may write

$$\begin{aligned}\text{Eddy loss}\quad&=aE^2.\\ \text{Hysteresis loss}\quad&=bE,\\ \text{Brush loss}\quad&=cE.\end{aligned}$$

where a, b, c are constants.

Then

$$\begin{aligned}a\,.\,100^2+b\,.\,100+c\,.\,100&=5\cdot5\times100\\ a\,.\,100^2+b\,.\,100+2c\,.\,100&=6\times100\\ a\,.\,150^2+b\,.\,150+2c\,.\,150&=7\cdot75\times100,\end{aligned}$$

whence we find:

$$\begin{aligned}a&=0\cdot035\\ b&=1\cdot5\\ c&=0\cdot5.\end{aligned}$$

At 125 volts, we have, then:

Eddy loss	$=0\cdot035\times125^2$	$=546$ watts.
Hysteresis loss	$=1\cdot5\times125$	$=188$,,
Brush loss	$=0\cdot5\times125$	$=62\cdot5$,,

3. A shunt dynamo gives 200 amps. at 500 volts and 800 R.P.M. The armature resistance under running conditions is 0·06 ohm, and the shunt resistance 300 ohms. The power taken to run the machine at full speed, not excited, and on open circuit, is 600 watts. The machine is then separately excited to its full excitation. The power taken to run it on open circuit at its full speed is 1800 watts, and at half speed the power required is 800 watts. From these observations deduce the full load efficiency of the machine, and obtain separately (1) the copper loss, (2) the Eddy Current loss, (3) the Hysteresis loss, (4) the Friction loss. (Mech. Sc. Trip. 1906.)

The 600 watts is the friction loss at full speed. If we take this to be proportional to the speed, at half speed it will be 300 watts.

At full speed, eddy loss + hysteresis loss $= 1800 - 600$,
$= 1200$ watts.
,, half ,, ,, ,, ,, $= 800 - 300$,
$= 500$ watts.

Let this combined loss $= an + bn^2$, where $n =$ speed in R.P.M.

Then
$$800a + 640{,}000b = 1200,$$
$$400a + 160{,}000b = 500,$$

whence
$$a = 1,\ b = \frac{1}{1600}$$

$\therefore$ at 800 R.P.M., hysteresis loss $= 800$ watts.

Eddy current loss $= \dfrac{1}{1600} \times 640{,}000 = 400$ watts.

At full load the shunt current $(I_s) = \dfrac{500}{300} = 1{\cdot}66$ amps.

$\therefore$ $I_a =$ armature current $= 201{\cdot}6$ A.

Copper loss for armature $= (201{\cdot}6)^2 \times 0{\cdot}06 = 2424$ watts.

Copper loss for shunt $= \dfrac{250{,}000}{300} = 833$ watts.

Hence (1) Copper loss $= 3257$; watts, (2) Eddy loss $= 400$ watts, (3) Hysteresis loss $= 800$ watts, (4) Friction loss $= 600$ watts. Total loss $= 5057$ watts.

$$\eta = \text{efficiency} = \frac{100{,}000}{105{,}057} = 94{\cdot}5\%.$$

4. A dynamo loses 200 watts in friction, 200 watts in hysteresis, and 300 in eddy currents. If its speed be increased 20% and its E.M.F. be increased 30% (due partly to the increased speed, partly to an increase in the excitation), find the new values of these losses. (Inter-Collegiate Exam., Cambridge, 1910.)

We have: Hysteresis loss $\propto n\mathcal{B}^{1\cdot 6}$

and eddy current loss $\propto n^2\mathcal{B}^2$.

Also
$$E \propto \phi n$$
$$\propto \mathcal{B}n.$$

$\therefore$
$$\mathcal{B} \propto \frac{E}{n}.$$

$\therefore$ Hysteresis loss $\propto n\left(\frac{E}{n}\right)^{1\cdot 6} \propto \frac{E^{1\cdot 6}}{n^{0\cdot 6}} = a\frac{E^{1\cdot 6}}{n^{0\cdot 6}}$ say.

and eddy current loss $\propto n^2\left(\frac{E}{n}\right)^2 \propto E^2 = b\,E^2$ say.

Then from the data, we have

$$a\,\frac{E^{1\cdot 6}}{n^{0\cdot 6}} = 200 \text{ and } b\,E^2 = 300..$$

and we have to find the value of $a\,\frac{(E\times 1\cdot 3)^{1\cdot 6}}{(n\times 1\cdot 2)^{0\cdot 6}}$ and $b\,(E\times 1\cdot 3)^2$.

The new hysteresis loss, then, $=\frac{(1\cdot 3)^{1\cdot 6}}{(1\cdot 2)^{0\cdot 6}}\,a\frac{E^{1\cdot 6}}{n^{0\cdot 6}}$,

$= 272$ watts;

and the new eddy current loss $\doteq (1\cdot 3)^2 bE^2$,

$= 1\cdot 69 \times 300$,

$= 507$ watts.

6. A machine is run at constant speed by a separate prime mover whose output is measured. When unexcited the machine takes 200 watts; when excited to give 100 volts it takes 700 watts; when excited to give 200 volts it takes 1910 watts. Find approximately the loss if the excitation is left at its last value, but the speed is increased till the machine gives 250 volts. (Inter-Collegiate Exam., Cambridge, 1911.)

Friction loss $=200$ watts in the first three cases.

Iron loss at 100 volts $=700 \quad -200=500$ watts.

" " " 200 " $=1910-200=1710$ "

The speed is constant, and the induction in the second case twice its previous value,

$\therefore$ we have

$$a(100)^{1\cdot 6}+b(100)^2= 500.$$
$$a(200)^{1\cdot 6}+b(200)^2=1710.$$
$$1600a+10{,}000b= 500.$$
$$4800a+40{,}000b=1710.$$
$$10{,}000b= 210.$$
$$b=\cdot 021.$$
$$a=\cdot 181.$$

Now $E \propto$ flux $\times$ speed.

$\therefore$ if the excitation is constant, $E \propto$ speed.

$$\therefore \quad \frac{\text{speed at 250 volts}}{\text{speed at 200 volts}}=\frac{250}{200}=1\cdot 25.$$

At n R.P.M. eddy current loss $=0\cdot 021 \times (200)^2$,

$=840$ watts.

$\therefore$ " $1\cdot 25n$ " " " " $=840 \times (1\cdot 25)^2$,

$=1320$ watts.

" n hysteresis loss $=\cdot 181 \times (200)^{1\cdot 6}$

$=870$ watts.

$\therefore$ " $1\cdot 25n$ " " $=870 \times (1\cdot 25)^{1\cdot 6}$,

$=1200$ watts.

The friction loss $\propto$ the speed.

$\therefore$ new friction loss $=1\cdot 25 \times 200=250$ watts.

$\therefore$ at 250 volts total loss $=250+1320+1200$,

$=2770$ watts.

7. In a certain coupled plant consisting of two identical machines, one machine is used as a prime mover, being worked as a motor with constant excitation and constant armature pressure. The speed of the pair is practically constant.

The ohmic loss in the motor's armature was negligible, a P.D. on it was 100 volts. The following results were obt the condition of excitation and armature circuit referr the driven machine, the current to the armature current motor :

Excitation.	*Armature.*	*Current.*
	open	13 amps.
to give 100 volts	,,	23 ,,
to give 50 volts	,,	15·9 ,,

Find for the driven machine the eddy current lo hysteretic loss at 100 volts excitation. (Mech. Sc. Trip.

The power taken in the three cases is 1300, 230c watts respectively.

The speed being constant, the frictional loss is cc Let it $=P_1$ for either machine.

The iron loss of the motor is constant, since the and excitation is constant, and $=P_2$ say.

Let P_3 = core loss of dynamo at 100 volts excitation

$P_3' =$,, ,, ,, 50 ,, ,,

Since n is constant, we can say

$$P_3 = a\mathscr{B}^{1\cdot 6} + b\mathscr{B}^2.$$

The induction in the second case will be $\frac{1}{2}\mathscr{B}$,

$$\therefore \qquad P_3' = a\left(\frac{\mathscr{B}}{2}\right)^{1\cdot 6} + b\left(\frac{\mathscr{B}}{2}\right)^2$$

We have then,

(i) $\ldots 2P_1 + P_2 = 1300$

(ii) $\ldots 2P_1 + P_2 + P_3 = 2300$ $\qquad \therefore \quad P_3 = 1000, \ P_3' =$

(iii) $\ldots 2P_1 + P_2 + P_3' = 1590$

$$a\mathscr{B}^{1\cdot 6} + b\mathscr{B}^2 = 1000$$

$$a\left(\frac{\mathscr{B}}{2}\right)^{1\cdot 6} + b\left(\frac{\mathscr{B}}{2}\right)^2 = 290.$$

Whence $a\mathscr{B}^{1\cdot 6} = 515$, $b\mathscr{B}^2 = 485$ watts. The former rep the eddy current loss, the latter the hysteretic loss at 1c excitation.

§ 125. **The Retardation Test.** Besides the above, ıer method of determining the losses is to run the machine › the highest safe speed, shut off the power, and let it slow ı, its own kinetic energy being used to overcome the losses. e the machine is slowing down (the armature being on circuit) the speed is read at vals, and so a speed-time curve tained; this is done for different ɜs of the field current. The est difficulty is reading the speed rately; of course if a self-recording ɔmeter be available, all is easy gh, otherwise the best method read the armature voltage every seconds, for with constant ex- ion the armature volts on open circuit are proportional e speed, and the relation between them can be found by ving the voltage at any particular speed.

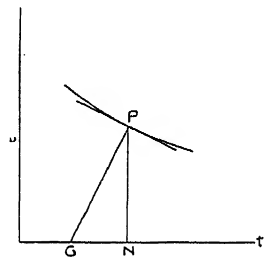


FIG 85

The following is the theory of the method :

Let I = moment of inertia of all the rotating parts.

ω = the augular velocity in radians per second.

P = the corresponding instantaneous loss, i.e. the rate of working.

M = the retarding couple.

Then
$$P = M\omega$$
$$M = I\frac{d\omega}{dt},$$
$$P = I\omega\frac{d\omega}{dt}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (49).$$

Now, let PG be the normal to the velocity-time curve, lraw PN perpendicular to the axis of time, then (Fig. 85), ıave

$$\text{PN} = \omega, \text{ and } \tan \text{G}\hat{\text{P}}\text{N} = \frac{d\omega}{dt}.$$

$$\text{the subnormal GN} = \omega\frac{d\omega}{dt}.$$

:e
$$P = I\omega\frac{d\omega}{dt} = I.\text{GN}.$$

That is, the instantaneous total loss is proportional to the length of the subnormal to the velocity-time curve. Therefore, if we plot the different values of GN against speed, for given exciting currents, we obtain a series of curves, which, to a certain scale, give the total iron and friction loss. By repeating the experiment with the field unexcited, we can find a curve of friction loss separately, and the difference between the ordinates of the two curves represents the iron loss.

The connection between the actual loss, P, in watts, and the length of subnormal in, say, inches, will be of the form

$$\mathrm{P} = a \,.\, \mathrm{GN},$$

where a is a constant involving the moment of inertia, which is difficult to estimate. We may, however, eliminate a by taking an extra retardation run with the machine loaded to p watts. We have then

$$P_1 = a \,.\, \mathrm{G_1N_1}$$

$$P_1 + p = a \,.\, \mathrm{G_2N_2},$$

where $\mathrm{G_1N_1}$ and $\mathrm{G_2N_2}$ are the subnormals to the two curves, at the same speed and the same excitation. Thus

$$a = \frac{p}{\mathrm{G_2N_2} - \mathrm{G_1N_1}},$$

and so the scale is determined.

§ 126. Since the E.M.F. is proportional to the speed when the excitation is constant we can write $\omega = \lambda E$, where λ is a constant, and the equation for P takes the form

$$P = I\,\omega\frac{d\omega}{dt} = I\lambda^2\frac{dE}{dt} \qquad (50).$$

Similarly the kinetic energy of the rotating parts can be expressed as $\frac{1}{2}I\omega^2 = \frac{1}{2}I\lambda^2E^2$.

§ 127. **Example.** In order to test a 5 *k*.W. 220 volt P.D. shunt dynamo a small motor was connected to it by means of a friction clutch. The dynamo field was separately excited with 220 volts and the machine run up beyond its normal speed. The times for the armature P.D. to fall from 222·5 to 217·5 volts, with the motor disconnected, were found to be (1) 106 seconds with no current in the armature, (2) 26 seconds with a mean current of 3 ampères in the armature. Taking

the armature resistance 0·04 ohm and the field resistance 260 ohms, find approximately the full load efficiency. (Mech. Sc. Trip. 1912.)

Here the mean value of E, over the observed range, is 220 volts, and, if P = the rotational losses, we have

$$P = I\lambda^2\frac{dE}{dt} = I\lambda^2\frac{222{\cdot}5 - 217{\cdot}5}{106} = I\lambda^2\frac{5}{106}.$$

And, when delivering 3 amps.,

$$P + 3 \times 220 = I\lambda^2\frac{5}{26}.$$

Hence, combining the two equations

$$P + 660 = P\frac{106}{26}.$$

$$\therefore \qquad P = 215 \text{ watts}.$$

To find the efficiency:

$$\text{The shunt current} = \frac{220}{260} = 0{\cdot}84 \text{ amps}.$$

$$\therefore \qquad \text{the shunt loss} = 0{\cdot}84 \times 220 = 186 \text{ watts}.$$

$$\text{The armature current} = \frac{5000}{220} = 22{\cdot}7 \text{ amp}.$$

$$\therefore \qquad \text{armature copper loss} = (22{\cdot}7)^2 \times 0{\cdot}04 = 26 \text{ watts}.$$

and the rotational losses we found = 215 watts.

$$\therefore \qquad \text{the total losses} = 186 + 26 + 215$$

$$= 427 \text{ watts},$$

$$\text{the output} = 5000 \text{ watts}.$$

$$\therefore \qquad \eta = \text{the efficiency} = \frac{5000}{5427} = 92{\cdot}2\%.$$

OPPOSITION TESTS.

§ 128. When we have two similar machines they may be conveniently tested by coupling them together mechanically and electrically in such a way that one works as a motor and drives the other as a generator, while this in turn supplies electrical power to the first, the power for the losses being

supplied by some external source. The power required for the losses may be supplied either electrically or mechanically, or by both methods.

§ 129. **Blondel's Method.** The copper losses are supplied electrically, whilst the iron and friction losses are supplied mechanically (Fig. 86). The two machines are connected by

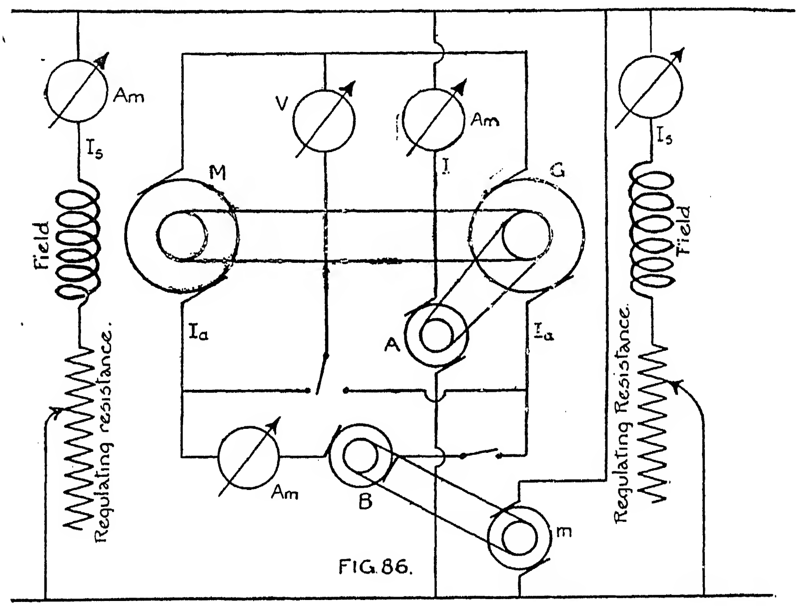


FIG. 86.

belts, or, preferably, direct-coupled, and the armatures connected so as to oppose one another. The fields are separately excited from the line. In Fig. 86 the machine M is working as a motor, and the other, G, as a generator; A is an auxiliary motor coupled to G to supply the power for the iron and friction losses; B is a booster (see Chap. VI.), driven by a motor *m*, which generates a voltage equal to the armature drop in the two armatures.

When both machines have their fields excited to the same amount, the electro-magnetic torque in each is the same, since they have the same speed and the same armature current. Hence there is no surplus power left between them for over-

coming the mechanical (iron and friction) losses, and these are entirely supplied by the motor A.

Again, since both machines have the same excitation and speed, and are similar machines, the E.M.F. induced in each will be the same. Hence the current flowing through the two armatures is that due to the voltage of the booster B. Hence we see that the iron and friction losses are supplied mechanically by the motor A, and the copper loss by the booster B.

Let I_s = the shunt current in each.
V = voltage of generator.
I_a = the current through the armatures.
V_b = voltage of booster armature.
I = the current taken by the auxiliary motor.

The losses in the motor A can be neglected in comparison with the power circulated between the main machines.

Then the useful output of generator $= V(I_a - I_s)$
the iron and friction loss of each $= \frac{1}{2}VI$
the copper ,, ,, $= \frac{1}{2}V_bI_a$
the shunt ,, ,, $= VI_s$.

$$\therefore \quad \text{efficiency} = \frac{V(I_a - I_s)}{VI_a + \frac{1}{2}VI + \frac{1}{2}V_bI_a}.$$

The efficiency of the motor will be

$$\frac{VI_a - \frac{1}{2}VI - \frac{1}{2}V_bI_a}{V(I_a + I_s)}$$

§ 130. **Hopkinson Test.** The booster set is omitted, and all the losses are supplied by the auxiliary motor, a current being caused to circulate by having one field slightly more strongly excited than the other. In consequence of the fields being at different excitations the iron losses in the two machines will be different, so that this method does not give the efficiency so accurately as Blondel's method since it is necessary to assume that the total loss is equally divided between the two machines, i.e. that the rotational losses are the same in each, for the copper losses are the same.

Let P_m = power supplied by auxiliary motor,

R_a = resistance of either armature.

Then total copper loss in both armatures $= 2I_a^2 R_a$.

$\therefore$ the rotational loss of each $= \frac{1}{2}P_m - I_a^2 R_a$.

The power taken by the shunt $= VI_s$.

The efficiency of the generator $= \dfrac{V(I_a - I_s)}{VI_a + \frac{1}{2}P_m}$,

and that of the motor

$$= \frac{VI_a - \frac{1}{2}P_m}{V(I_a + I_s)},$$

§ 131. **Hutchinson's Method.** We deal now with opposition tests in which all the losses are supplied electrically. The most accurate is Hutchinson's test, in which the copper

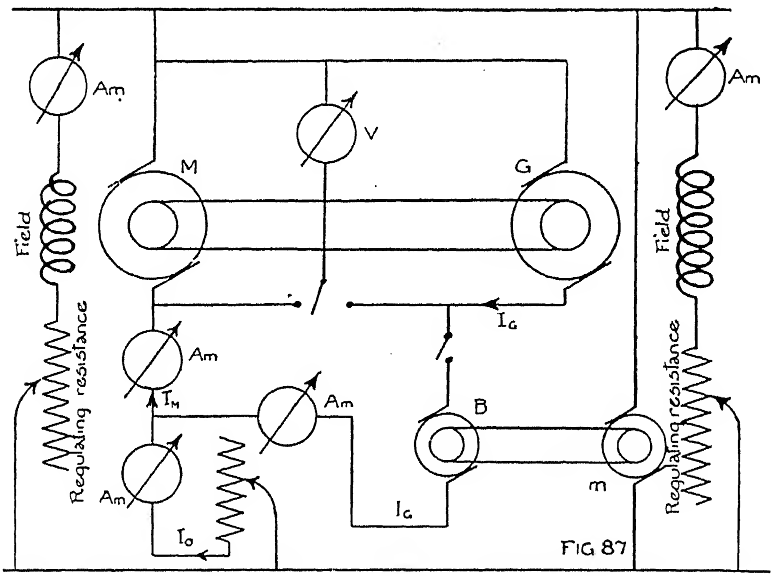


FIG 87

loss is supplied by a booster (see Chap. VI.), and the other losses by the line. If the booster is omitted we have Kapp's test, all the losses being supplied from the mains.

The connections are shown in Fig. 87, where M and G are the two similar machines under test, the former working as a

motor, and the latter as a generator. B is the booster and *m* the booster motor.

Let V_o = pressure across mains.

V = pressure across terminals of M.

I_o = current from mains, excluding that taken by the shunts.

I_g = current from booster.

I_m = ,, taken by M.

P = joint rotational losses of M and G.

Then the total power supplied by the line and the booster is

$$V_o I_o + V I_g.$$

The total copper loss is $I_g^2 R_a + I_m^2 R_a$, where R_a is the resistance of either armature. Hence we have

$$V_o I_o + V I_g = (I_g^2 + I_m^2) R_a + P.$$

The fields of the two machines must be equally excited, so that they may have the same iron losses. This being the case the voltage of the booster must equal the drop in the two armatures, i.e.

$$V = (I_g + I_m) R_a.$$

Also, we have $$I_o = I_m - I_g.$$

Hence
$$\begin{aligned} P &= V_o I_o + V I_g - (I_g^2 + I_m^2) R_a, \\ &= V_o I_o + I_g (I_g + I_m) R_a - (I_g^2 + I_m^2) R_a, \\ &= V_o I_o + I_g I_m R_a - I_m^2 R_a, \\ &= V_o I_o + R_a I_m (I_m - I_o) - I_m^2 R_a, \\ P &= I_o (V - I_m R_a). \end{aligned}$$

Then the rotational losses in each machine are $\frac{1}{2}P$.

§ 132. **Kapp Test.** (Sometimes called the Kapp form of the Hopkinson Test.) This only differs from the last in so far as all the losses are supplied from the line, the booster being omitted. This method is not so accurate as Hutchinson's, as it entails assuming the iron losses the same in each machine, which is not true, since both have the same P.D. and

speed, but one is working as a motor and the other as a generator, so that their induced E.M.F.'s and therefore their excitations, must differ. The connections are shown in Fig. 88.

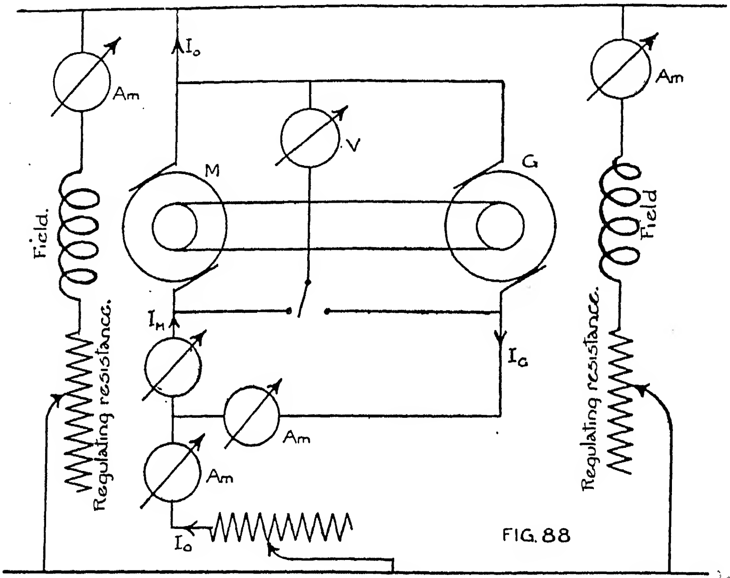


FIG. 88

Let V =pressure of supply from mains.

I_o=current supplied from mains, excluding shunts.

I_m=current in motor's armature.

I_g= ,, ,, generator's ,,

R_a=resistance of either armature.

Then the total power supplied to both armatures is VI_o.

The copper loss in M's armature$=I_m{}^2R_a$.

,, ,, ,, G's ,, $=I_g{}^2R_a$.

∴ the total rotational losses of the two machines are

$$P=VI_o-(I_m{}^2+I_g{}^2)R_a;$$

and that in each machine is $\frac{1}{2}P$.

Also $$I_m=I_o+I_g$$

Then we have, for the efficiency of each, if I_{mf} and I_{gf} are the field currents, and with the same armature current under working conditions as under test conditions,

	Motor.	*Generator.*
Rotational losses	$\frac{1}{2}P$	$\frac{1}{2}P$
Armature loss in copper ..	$I_m{}^2R_a$	$I_g{}^2R_a$
Shunt loss	VI_{mf}	VI_{gf}
Output	input − losses	$V(I_g - I_{gf})$
Input	$V(I_m + I_{mf})$	output + losses.

Hence $$\eta_m = \frac{VI_m - (\frac{1}{2}P + I_m{}^2R_a)}{V(I_m + I_{mf})}$$

and $$\eta_g = \frac{V(I_g - I_{gf})}{VI_g + (\frac{1}{2}P + I_g{}^2R_a)}.$$

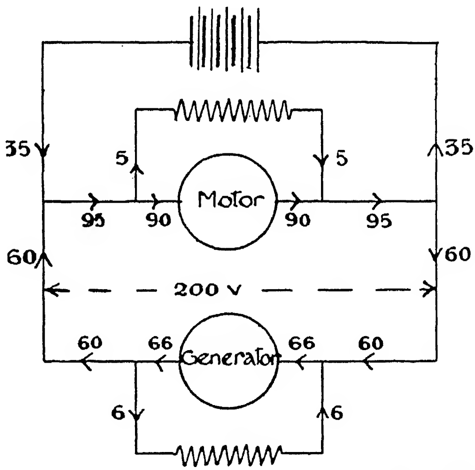


FIG 89

§ 133. **Example.** In a Kapp form of the Hopkinson test the following results were obtained :

Constant P.D.	= 200 volts.
Current supplied from outside sources	= 35 amps.
Resistance of each armature	= 0·08 ohm.
Total motor current	= 95 amps.
Motor field current	= 5 amps.
Dynamo field current	= 6 amps.

Find the efficiency of each machine. (Mech. Sc. Trip. 1910.)

Motor.	Total current	=95 amps.
	Field ,,	= 5 ,,
∴	Armature ,,	=90 ,,
	Again, total current	=95 ,,
	Supply from outside	=35 ,,
∴	,, ,, generator	=60 ,,
	Generator's field current	= 6 ,,
∴	,, armature ,,	=66 ,,

Energy lost in the two armatures $=(90-66)\times 200$ $=4800$ watts

I^2R loss in generator's armature $=(66)^2\times 0{\cdot}08$ $=349$,,

I^2R loss in motor's armature $=(90)^2\times 0{\cdot}08$ $=648$,,

∴ rotational loss in the two machines

$=4800-349-648=3803$,,

∴ rotational loss in each machine $=1901$,,

Motor.

	Core loss	=1901 watts.
	Shunt loss	$=200\times 5=1000$ watts.
	Armature I^2R loss	= 648 ,,
∴	Total loss	=sum of these.
		=3549 watts.
	Current supplied	=95 amps.
	Energy supplied	$=95\times 200=19{,}000$ watts.
	Energy delivered	$=19{,}000-3549=15451$ watts

$$\therefore \qquad \eta=\frac{15{,}450}{19{,}000}=81{\cdot}3\%$$

Generator.

	Cores loss	=1901
	Shunt loss	=1200
	Armature I^2R loss	= 349
∴	Total loss	=3450
	Current delivered	= 60 amps.
	Energy ,,	=12,000 watts.
	Energy supplied	=15,450 ,,

$$\eta=\frac{12{,}000}{15{,}450}=77{\cdot}6\%.$$

SEPARATION OF LOSSES IN SERIES MOTORS.

§ 134. In a series motor the speed, exciting current and ad are all intimately connected so that the results obtained : no-load with normal connections are useless for determining ie efficiency at any other load, and the following is one of the est methods for testing series motors :—

(1) **Determination of Iron and Friction Loss together.** he motor is connected up as shown in Fig. 90, the armature eing shunted by a resistance, so that the field current and rmature current may be varied independently. The armature run light so that its copper loss is negligible and the power iken by it represents the total iron and friction loss correspond-ig with the condition f speed and excitation ien obtaining. Now or a given excitation ie speed is propor-onal to the armature .M.F., so that we an say the iron losses

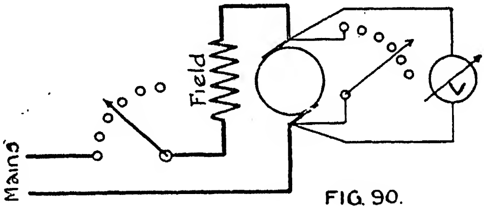


FIG. 90.

epend on field current and armature E.M.F., the latter being early equal to the P.D. for light loads. By keeping the rmature volts constant a curve may be drawn showing the ron and friction loss for various field currents for a particular alue of the E.M.F., and so for other values of the E.M.F.

(2) **Determination of Friction alone.** The motor is onnected up to a source of supply and run as a series motor, nloaded, in the ordinary way, but the applied pressure must e kept very small to prevent the speed rising too high. With his small voltage the iron loss will be negligible, and we sup-ose the whole power taken by the armature to represent riction loss.

Then, by comparing the results with those obtained from 1) above, for the same excitation and armature E.M.F., we can eparate the friction loss from the iron loss. To separate the atter into its Hysteresis and Eddy components for the given xcitation we can obtain, in the above manner, the iron losses vith that excitation corresponding with two different E.M.F.'s., nd then proceed as in Example (2) § 124.

§ 136. **Field's Method.** This test is used mostly for traction motors, and the motors are connected by gearing designed to represent, as nearly as possible, the gears used under working conditions, so that the gear losses are included in the test. Two similar motors are geared together so as to run at the same speed, and one works as a motor and the other as a generator. The current from the mains passes through the motor M and excites the field of the generator G,

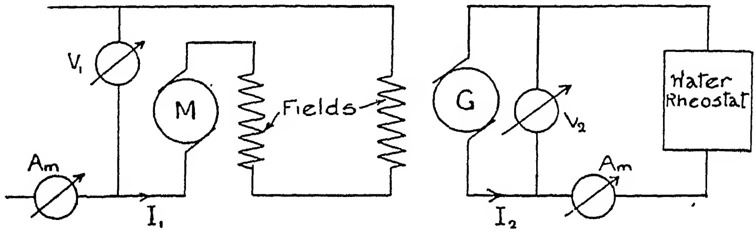


FIG. 91.

thus both machines have the same speed and same excitation, so that the losses due to iron and friction are the same in both. The generator is loaded by a water resistance.

Let r = resistance of motor and two fields.

R_a = ,, ,, generator's armature.

The total loss due to iron and friction

$$=(V_1I_1-V_2I_2)-(I_1^2r+I_2^2R_a),$$

and one half of this belongs to each machine.

EXAMPLES.

1. The following particulars refer to a 5 H.P. motor, whose armature is wave wound:

Number of conductors	760.
Mean length of one turn	70 cm.
Cross section of conductor	0·02 cm.
Specific resistance	$2\cdot10^{-6}$ ohm.
Full load current to armature	17·5 A.

Calculate the full load copper loss in the armature.

2. A motor is designed to take a full load current of 120 A. at 300 V. The field resistance is 100 ohms, and the armature resistance is 0·08 ohm. From the observations given below deduce (i) the full load efficiency, (ii) the loss due

to hysteresis, (iii) the loss due to eddy currents. (In deducing the two latter neglect armature drop.)

Field separately excited with 300 V.

$$\left.\begin{array}{lll}\text{Armature current at } 300 \text{ V.} & = 4 \text{ A.} \\ \quad\text{,,} \quad\quad \text{,,} \quad\quad \text{,,} \quad 200 \text{ V.} & = 3{\cdot}3 \text{ A.}\end{array}\right\} \text{at no-load.}$$

Total friction loss = 300 W.

Speed at 300 V. = 300 R.P.M.

(Inter-Coll. Exam., Cambridge, 1906.)

3. An armature is running at a fixed speed, and loses 400 W. when giving 200 V. E.M.F., $\frac{1}{4}$ of this is due to friction, and $\frac{1}{4}$ to eddy currents. The excitation is altered to give 300 V. What will be the new loss? (Inter-Coll. Exam., Cambridge, 1913.)

4. A shunt machine was excited with 200 V.; with an armature pressure of 200 V. it took 5 A. in the armature to run light, and 4·5 A. at 150 V. Its shunt resistance was 40 ohms, and its armature resistance 0·05 ohm. When the machine is taking 150 A. at 200 V. as a motor, deduce the efficiency, and allocate the various losses as far as possible. (Inter-Coll. Exam., Cambridge, 1914.)

5. A separately excited D.C. motor, when run without load, at 500 V., takes a current of 1 A. With the same excitation, but with 250 V. on the armature, the no-load current is 0·8 A. How much power is wasted in hysteresis and friction, and how much in eddy currents, when the motor is running at 500 V.? (Mech. Sc. Trip. 1914.)

6. For test purposes the field of a shunt-wound dynamo is separately excited, and the E.M.F. of the armature on open circuit is read by a voltmeter. The following three tests and observations were made:

	Field.	*Volts in armature on open circuit.*	*R.P.M.*	*Power taken to drive as a motor, watts.*
1	not excited	0	400	360
2	excited	300	600	1500
3	excited	400	400	1300

From these data deduce the eddy current and hysteresis losses on open circuit at 500 V. and 600 R.P.M. (Mech. Sc. Trip. 1911.)

7. A compound wound D.C. generator, whose full load output is 100 A. at 500 V., is tested by coupling it to a small motor and measuring the power required to run it, as follows :—

(*a*) Armature and series turns short-circuited through an ammeter. Shunt current zero. Armature current 75A. Power taken 1·5 *k*w.

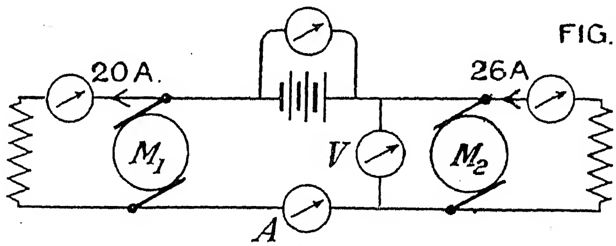
(*b*) Armature circuit open. Shunt current 1·8 A., machine giving 500 V. Power taken 3·9 *k*w.

(*c*) Armature and shunt circuits both open. Power taken 0·6 *k*w.

The losses in the motor have been deducted in each case. Calculate the efficiency of the machine at full load, and the resistance of the armature circuit. (Mech. Sc. Trip. 1907.)

8. Fig. 92 shows a method of carrying out the Kapp test of a pair of shunt machines. Assuming the *total* losses

FIG. 92



to be equally divided between the two machines, find the efficiency of each when the ammeter A reads 400 A. The P.D. across the battery terminals is 120 V., and the voltmeter V reads 600 V. In the diagram M_1 is working as a motor driving M_2 as a generator. (Mech. Sc. Trip. 1913.)

CHAPTER V.

APPLICATION OF D.-C. MOTORS TO TRACTION.

§ 137. It is not proposed to enter into the general problem of electric traction, for which the reader is referred to works on this subject, but to treat shortly the motor equipment of trams and railways.

§ 138. It is usual to employ two motors, controlled on the series-parallel system, but occasionally four are employed. The motors are, almost without exception, series wound and totally enclosed, and they must possess the ability to run in either direction without alteration being made to the position of the brushes, i.e. with the brushes fixed permanently in the position of symmetry, which is made possible by the use of carbon brushes. It is also essential that with the brushes in this position, the number of commutator segments should be large (to reduce the number of turns shorted between a pair of segments), and that the ratio of field ampère-turns per pole to armature ampère-turns per pole should be large; usually it is not much less than two. The purpose of this is to minimise field distortion by armature reaction, thereby partly fulfilling the conditions for sparkless commutation with the brushes in a symmetrical position.

Before considering the performance of D.-C. motors when employed for electric traction, we shall treat briefly the mechanics of train haulage.

§ 139. **Train Resistance.** The total resistance to trains in motion consists of (1) Atmospheric resistance, (2) Journal friction, track resistance, and so on.

Atmospheric resistance is caused partly by end resistance, and partly by skin friction, the former being given approximately by $0{\cdot}0028v^2$ pounds per square foot, where v is the

velocity in miles per hour.* For skin friction it is usual to add about 10 or 15 per cent. of the above for each coach except the front one, but various formulae have been suggested for its computation.

The second part of the resistance is made up of journal and gear friction, friction between the wheel flanges and the rails, and the effects of unevenness of track. It is not open to exact calculation, and is generally assumed as proportional to the weight of the train or tram, being expressed as so many pounds per ton, or as equivalent to a certain up gradient.

§ 140. **Power required.** Suppose we wish to run a train of total weight w tons at v miles per hour on a level track, the total resistance being equivalent to k lbs. per ton. What horse power is necessary?

At a uniform speed of v miles per hour the rate of working is

$$kw \times \frac{22v}{15} \text{ft.lbs. per second,}$$

so that the horse-power required is

$$\text{H.P.} = \frac{22kwv}{15 \times 550} = \frac{\text{kwv}}{375} \quad \ldots\ldots\ldots\ldots \quad (51).$$

Next, what is the acceleration with a given tractive force? (The maximum acceleration is limited by the coefficient of adhesion, i.e. the proportion of the total weight which can be applied to the wheels without causing them to slip, for this is the greatest tractive effort whch can be employed.) Now suppose that F is the tractive effort, expressed in pounds. The nett effective force is F—kw pounds; hence the acceleration (f) is given by

$$\frac{F-kw}{2240w} = \frac{f}{g}$$

$$\text{or } f = \frac{32 \cdot 2(F-kw)}{2240w} = \frac{F-kw}{70w}, \text{ nearly} \ldots. \quad (52).$$

* *Proc. Inst. C.E.*; Vol. 147.

On a gradient the resistance due to gravity must be included in the total resistance, being expressed by $2240w\frac{s}{100}$, or $22{\cdot}4s$ lbs. per ton of train weight, where s is the percentage gradient, i.e. the rise in feet per 100 feet of track.

§ 141. **Effect of Inertia of Rotating Parts.** In the above expression for the acceleration we have omitted the flywheel-effect of the rotating parts, namely, the wheels and armatures. Let us now see what effect this has on the acceleration.

Let w tons	=wt. of train.
w_1 ,,	=total weight of all wheels and axles.
k feet	=radius of gyration ,, ,,
w_2 tons	=total weight of all armatures.
k_2 feet	=radius of gyration of ,,
ω radians/sec.	=angular velocity of wheels.
$m\omega$,,	= ,, ,, motor.
v feet/sec.	=velocity of train.
D feet	=diameter of driving wheels.

Then the total kinetic energy

$$= \frac{1}{2g}(w_1 k_1{}^2\omega^2 + w_2 k_2{}^2 m^2\omega^2 + wv^2) \text{ ft. tons}$$

$$= \frac{v^2}{2g}\left(w_1\frac{4k_1{}^2}{D2} + w_2\frac{4k_2{}^2m^2}{D^2} + w\right)\text{ft. tons,}$$

which shows that the effect of the rotation is virtually to increase the weight of the train by

$$w_1\frac{4k^2}{D^2} + w_2\frac{4k_2{}^2m_2{}^2}{D^2} \text{ tons,} \qquad \ldots\ldots\ldots\ldots(53).$$

so far as questions of acceleration are concerned. The above quantity generally works out at about 7 to 10% of the train weight.

PERFORMANCE CURVES OF D.-C. TRACTION MOTORS.

§ 142. We shall consider now the performance of D.-C. motors, and show how to obtain the performance curves of a given design, that is to say, the curves of speed, B.H.P., tractive effort and efficiency, plotted against current.

The first curve required is the Saturation Curve, which can be calculated from the $\mathscr{B}-\mu$ curve of the iron employed, or plotted from an open-circuit test (§ 58), and so will be assumed known.

§ 143. **The Speed Current Curve** can be obtained directly from this, as in Examples 1 or 3, § 103.

§ 144. **B.H.P. Torque and Efficiency Curves.** To calculate these we must estimate the various losses in the machine (see Chapter IV.), and subtracting the total loss from the input in watts, we have the nett output, which, divided by 746, gives the B.H.P. From this and the speed we can calculate the torque.

Otherwise we can assume a certain efficiency, and calculate the gross torque from the formula (p. 25, if $m=1$) :—

$$\text{Torque in lbs.ft.} = \cdot 117 \times (p\Phi z 10^{-8}) \times I,$$

the saturation curve telling us the flux for any given current. From this gross torque and the assumed efficiency we can calculate the nett shaft torque, and hence the B.H.P.

In any case we have the relations

$$\text{B.H.P.} = \frac{2\pi n \times \text{torque}}{550} = \frac{\text{watts input} - \text{watts lost}}{746}$$

and

$$\frac{\text{B.H.P.} \times 746}{\text{watts input}} = \text{efficiency},$$

n being the speed in revolutions per second.

If the armature be mounted direct on the spindle of the driving wheels, we find the pull from the equation

$$\underset{\text{(lbs.)}}{\text{Tractive effort}} \times \underset{\text{(ft.)}}{\text{radius of wheel}} = \underset{\text{(lbs.ft.)}}{\text{torque}}.$$

§ 145. **Effect of Gearing.** The effect of gearing is allowed for by assuming it to be equivalent to a certain reduction of the motor efficiency, say about 5%. The tractive effort at the rail, and the speed of the car are then deduced as follows: Let r be the gear ratio, then the motor runs r times as fast as

the car wheels. Let D feet be the diameter of the driving wheels, v the speed of the car in miles per hour. Then

$$\frac{22}{15}v = \frac{n\pi D}{r}$$

or
$$v = \frac{15\pi n D}{22\ r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

and
$$\text{tractive effort} = \frac{r \times \text{motor torque}}{\frac{1}{2}D} \quad \ldots\ldots\ldots (55).$$

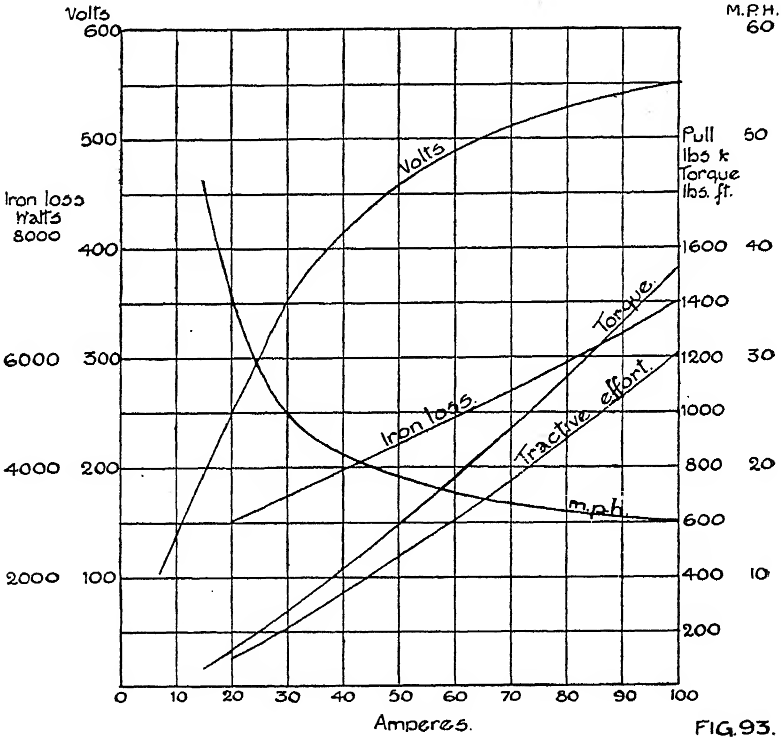


FIG. 93.

§ 146. **Example.** Let us now apply these principles to a numerical case:

Four-pole, 45 H.P. Traction Motor for 550 volts.
Resistance of Armature ....0·28 ohm.
,, ,, Field Coils ....0·29 ,,

The Saturation Curve, iron and friction losses are shown in Fig. 93, supposed known.

Then, neglecting brush resistance, the total resistance is 0·57 ohm, and we can proceed to draw up a table as follows:

Current (I)	15	20	30	40	50	60	70	80	90	100
Drop ($=0·57I$)	8·5	11·4	17·1	22·8	28·5	34·2	39·9	45·5	51·3	57·0
$E=V-$drop.	541·5	538·6	532·9	527·2	521·5	515·8	510·1	504·5	498·7	493·0
$E'=$induced E.M.F. at 750 R.P.M.	200	255	360	420	455	490	510	530	540	550
$n=$R.P.M. $=\frac{E}{E'}\times 750$	2030	1580	1110	940	860	785	750	715	690	670
Losses: Copper loss (*kw.*) $=0·57I^2$	0·127	0·228	0·513	0·912	1·42	2·05	2·79	3·64	4·62	5·7
Iron loss (*kw.*)	2·80	3·00	3·4	3·97	4·20	4·95	5·40	5·90	6·40	7·0
Total loss	2·93	3·23	3·91	3·97	5·62	7·00	8·19	9·54	11·02	12·7
Input (*kw.*) $=550I$	8·25	11·00	16·50	22·00	27·50	33·0	38·5	44·0	49·5	55·0
Output	5·32	7·77	12·59	17·12	21·88	26·0	30·2	34·5	38·5	42·3
B.H.P.	7·2	10·4	16·9	22·9	29·3	36·2	40·5	46·3	51·8	56·7
Efficiency %	65	70·6	76·4	78	79·5	79	78·5	78·5	77·6	76·9
Shaft torque (lbs.ft.) $=\frac{33,000\times\text{B.H.P.}}{2\pi n}$	18·7	34·6	80	128	179	243	284	340	395	443

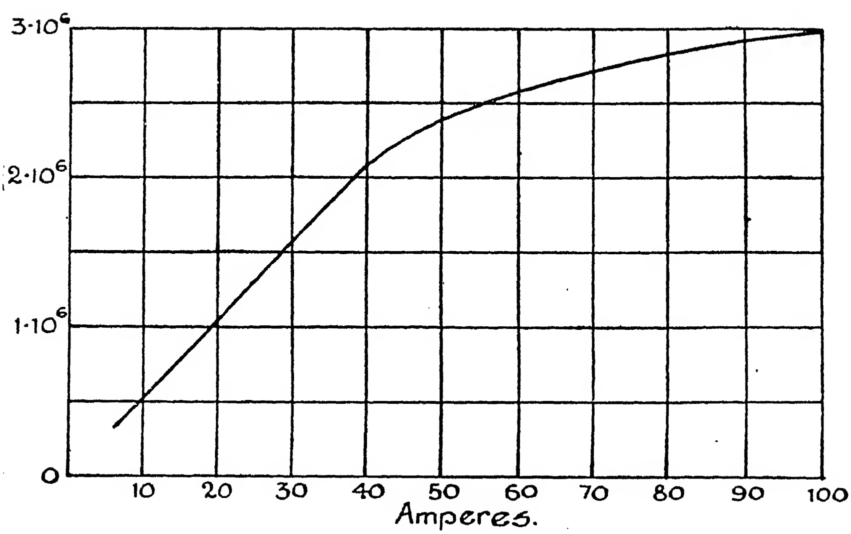
Now, suppose this motor be mounted on a car, being geared to the driving wheels with a ratio 4 : 1, the driving wheels having a diameter of 30″. Suppose the efficiency of the gear to be 85%. Then we can draw up a set of curves for the car, from the following table, in which M is the shaft torque :—

I	M' = Torque on driving wheels $=0{\cdot}85\times 4M$	Tractive effort $=\frac{M'}{1{\cdot}25}$ lbs.	n' = R.P.M. of wheels $=\frac{n}{4}$	v (M.P.H.) $=\frac{15}{22}\,\frac{\pi\times 2{\cdot}5\times n'}{60}$
15	63·5	50·8	510	45·5
20	128	102	395	35·2
30	272	218	276	24·6
40	435	347	235	21
50	610	488	215	19·2
60	825	660	196	17·5
70	965	770	187	16·7
80	1150	850	179	16
90	1340	1070	172	15·5
100	1510	1200	167	14·9

These curves are shown in Fig. 93.

§ 147. We will now take the same problem with slightly modified data. Fig. 94 shows the saturation curve of a four-pole motor for 550 volts. The following are further particulars of the design:

FIG 94



Armature winding	Two circuit.
Number of conductors	660.
Armature resistance..........	0·28 ohm.
Flux per pole	shown by curve.
Field resistance	0·29 ohm.
Full-load current	70 ampères.
Assumed efficiency	88 per cent.

In this case we proceed as follows: For any given current we find the drop, and hence the back E.M.F., then the speed is given by

$$n = \frac{60 E 10^8}{2 \Phi \cdot 660} \text{ R.P.M.}$$

The B.H.P. is found from the input and the assumed efficiency of 88%; from this and the speed the torque is deduced as before. Proceeding in this way we obtain the following values:

Current	20	30	40	50	60	70	80	90	100
Back E.M.F. ..	539	533	527	521	516	510	504	498	492
Flux Φ (megalines)	1·05	1·56	2·1	2·4	2·6	2·75	2·85	2·95	3·0
r.p.m. (n) ...	2320	1550	1140	985	900	842	800	765	740
Input (*k*w.) ..	11	16·5	22	27·5	33	38·5	44	49·5	55
Output at 88% *efficiency* (*k*w.).	9·7	14·5	19·4	24·2	29	34	38·7	43·6	48·5
B.H.P.	13	19·4	26	32·5	39	45·6	52	58·5	66
Torque (ft.lbs.)	29·4	65·5	118	173	228	285	340	400	468

With the motor geared 4 : 1 to driving wheels 30″ diameter, we obtain, in the same way as before the tractive effort and car velocity:

Current	20	30	40	50	60	70	80	90	100
Miles per hour.	51·8	44·5	25·4	22	20	18·7	17·8	17	16·4
Tractive effort (lbs.)	79	178	329	470	620	770	920	1090	1260

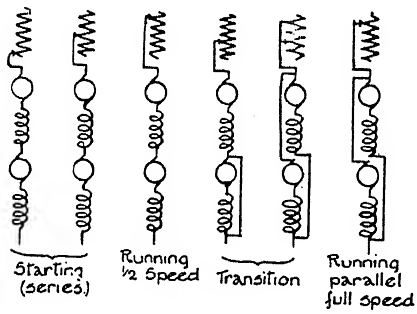
SPEED CONTROL OF CARS FITTED WITH SERIES MOTORS.

§ 148. **Series Parallel Control.** Electric cars and locomotives are usually fitted with two, or sometimes four, motors, which can be connected in series or in parallel. With two motors, when they are in series, each will have only half the line volts, which means that when exerting a given torque they will each run at about half the speed that they would have

under the full line pressure, for this reason: With a given torque, the current, and therefore the flux, will have a definite value, so that the speed will simply vary as the back E.M.F., which, with the motors in series, will be about one-half of what it would be when each is subjected to the full pressure.

On a car equipped with two motors the latter are connected in series at starting, with some resistance in circuit; this is gradually cut out, and when greater speed is required the motors are put in parallel, some resistance being again put in temporarily in order to prevent a sudden rush of current when each motor has the full line voltage. This method of control is called *series-parallel control*, and is almost universally used on cars driven by D.-C. series motors. A diagram of the successive connections for two motors is shown in Fig. 95.

FIG. 95



When four motors are employed they are often arranged in two pairs, the members of each pair being permanently connected in parallel, so that the same diagram will apply if a pair of motors connected in parallel be substituted for each motor.

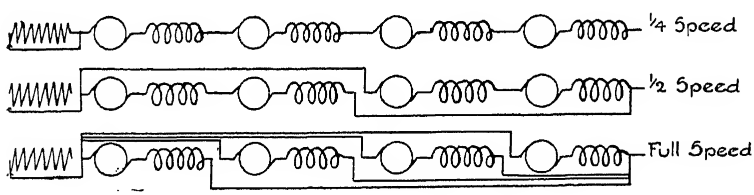


FIG 96

With four motors the wiring may be arranged so as to obtain three running speeds, a quarter, half, and full speed respectively, as shown in Fig. 96. The chief objection to this method is the greater complication in the car winding.*

* For detailed treatment of series-parallel control see *Experimental Electrical Engineering*, Karapetoff, Ch. 31, and *Dynamo and Motor Switchboard Circuits*, Bowker, Section 4.

§ 149. **Field Control for Series Motors.** One way to increase the speed of a motor is to weaken the field, and this led to sparking at the brushes before interpoles were introduced. With the use of interpoles, however, this disadvantage is overcome, and field weakening combined with series-parallel control is coming into use again. The operation is performed as follows: Each motor is provided with (say) four field coils which can be connected in series, series-parallel, or parallel;

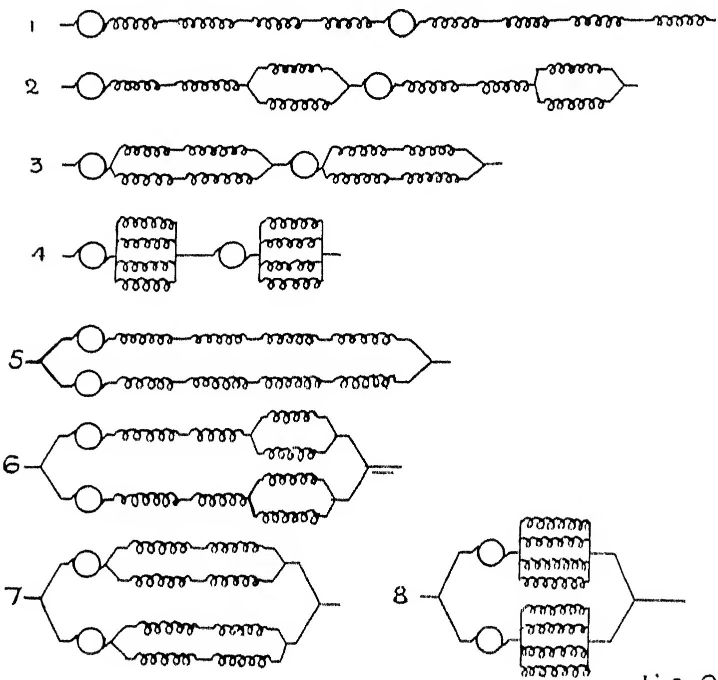


FIG. 97.

the motors are brought to the full speed series condition, with full field strength, in the usual way, and the field is then weakened by connecting the field coils in series-parallel and then in parallel, as in Fig. 97, and then follow similar combinations with the motors in parallel. In Fig. 97 the commutating fields are omitted.

It should be noted that the field must not be weakened to such an extent that the current required to produce a given torque will burn out the armature.

§ 150. **To calculate the performance of a Car equipped with two Motors, given tests of one Motor.** The test data of the motor are usually reduced to two curves, one connecting torque and current, and another giving the relation between speed (R.P.M.) and current for a given terminal P.D. If we know the diameter of the car wheels and the gear ratio and efficiency we can reduce the torque to tractive effort, and the speed to miles per hour. From these curves we can deduce a speed-current curve for the car with the motors in series or parallel.

Since E is proportional to the product Φn, it will also be proportional to the product Φv, where v is the speed of the car in miles per hour, and we can write

$$E = \Psi v. \qquad \qquad (56).$$

The factor Ψ depends only on the flux from the field magnets, and flux depends only on the current through the motor. Hence Ψ depends only on the current I, and is the same for any given value of I under all conditions.

If V_0 = P.D. applied at the test,

R = the resistance of the motor,

we have $$E = V_0 - IR.$$

Hence E can be calculated, and then $\Psi = \frac{E}{v}$, so that we can draw a curve of Ψ and I.

If the motors are in series, with a resistance r in circuit, we have

$$V = 2E + I(2R + r), \qquad \qquad (57)$$

or $$V = 2\Psi v + I(2R + r). \qquad \qquad (58)$$

With the motors in parallel, on a resistance r, the equation is

$$V = E + IR + 2I \,.\, r, \qquad \qquad (59)$$

or $$V = \Psi v + I(R + 2r). \qquad \qquad (60)$$

where I is the current taken by each motor, and V is the line voltage. Hence, if we know r, we can calculate the speed for any value of the current, or conversely we can find the resistance necessary for any required speed.

§ 151. **Example.** The curves below (Fig. 98) are from a test of a series motor. A tramcar weighing 12 tons is fitted with a pair of these motors, and has a tractive resistance of 36 lbs. per ton at all speeds. It is ascending an incline of 1 in 30 with a P.D. of 500 volts; at first with the motors in series with no extra resistance, until the speed becomes steady; and

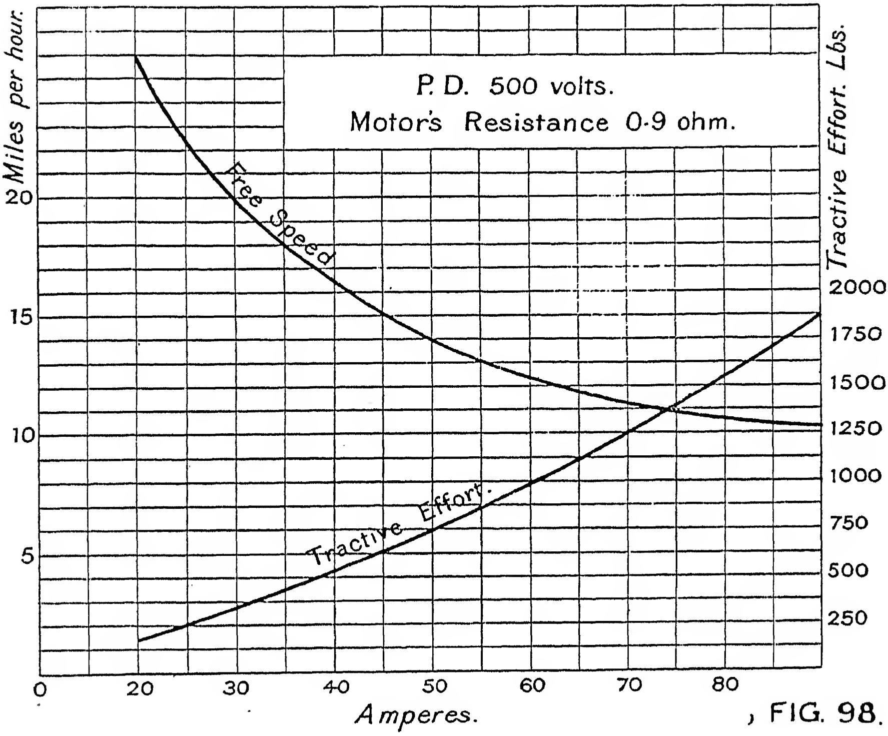


FIG. 98.

afterwards, with the motors in parallel with extra series resistance. Find the following:

(*a*) The steady speed attained with the first arrangement.

(*b*) The extra series resistance required if the sudden change from the series to the parallel arrangement is to produce no sudden change in the current of each motor.

(*c*) The steady speed attained with the parallel arrangement, with no extra resistance, and P.D. $=470$ volts.

(Mech. Sc. Trip. 1912.)

On an incline of 1 in 30 the pull required for a steady speed is

$$12 \times 2240 \times \frac{1}{30} + 12 \times 36$$

$$= 1326 \text{ lbs., i.e. } 663 \text{ lbs. per motor.}$$

From the curves we see that the current required is 46 amps. per motor, and that the corresponding speed, under test conditions, is 14·8 M.P.H.

Also $\qquad E = V - IR = 500 - 46 \times \cdot 9 = 458 \cdot 6.$

Hence, for $\qquad I = 46,$

$$\Psi = \frac{E}{v} = \frac{458 \cdot 6}{14 \cdot 8} = 31.$$

(*a*) With the motors in series, with no extra resistance,

$$V = 2E + I \,.\, 2R,$$

$$500 = 2E + 46 \times 1 \cdot 8.$$

$\therefore \qquad E = 208 \cdot 5.$

$\therefore \qquad v = \frac{E}{\Psi} = \frac{208 \cdot 5}{31} = 6 \cdot 74$ miles per hour.

(*b*) With the motors in parallel on a resistance r, and the same current (46 amps.) flowing through each,

$$V = E + I(R + 2r).$$

Also, since, for the moment, the speed as well as the exciting current is unaltered, E will be the same as before. Hence we have $\qquad 500 = 208 \cdot 5 + 46(\cdot 9 + 2r),$

$$r = 2 \cdot 72 \text{ ohms.}$$

(*c*) With the motors in parallel, with no extra resistance, the P.D. being 470 volts, we have

$$470 = E + 46 \times \cdot 9.$$

$\therefore \qquad E = 428 \cdot 6,$

with $\qquad I = 46, \; \Psi = 31.$

$\therefore \qquad v = \frac{E}{\Psi} = \frac{429}{31} = 13 \cdot 9$ miles per hour.

It should be noticed here that, in order to calculate the steady speed, we have assumed the same current in both cases. This is because the pull demanded is constant at a constant speed on a constant gradient.

§ 152. **Braking by the Motors.** If the motors be disconnected from the line and short circuited, the connections being rearranged so that the relative directions of the currents in the armature and field are reversed, they will run as series generators, being driven by the car. The kinetic energy of the car is utilised to drive the machines, and the electric power so generated is dissipated in heating the resistances. The connections for braking are shown in Fig. 99, in which AB is an equalizing connection to prevent the possibility of the current circulating around the two motors instead of through the resistance r.

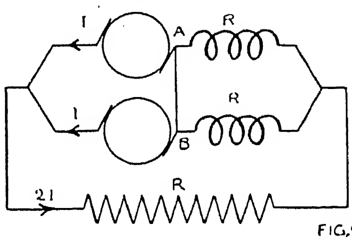


FIG. 99.

The electrical equation in this case is

$$\Psi v = E = I(R+2r), \quad \ldots\ldots\ldots\ldots\ldots\ldots (61).$$

and Ψ has the same value, for a given value of I, as found from the motor test curves, since, with a given I, there will be a given flux whether the machines be working as motors or generators.

Let P_m = pull developed by the machine as a motor, and

P_d = pull required to drive the machine as a generator at the same speed, and delivering the same current.

W_r = the rotational losses per machine.

Working as a motor we have

$$P_m \times v \propto \text{nett output},$$

$$= (EI - W_r) \times \text{a constant, C say.}$$

Working as a dynamo the equation becomes

$$P_d \times v = \text{total input},$$

$$= (EI + W_r) \times \text{C}.$$

Hence

$$\frac{P_d}{P_m} = \frac{EI + W_r}{EI - W_r} \quad \ldots\ldots\ldots\ldots\ldots (62).$$

If we write

$$\eta_1 = \frac{\text{net output as motor}}{\text{total input}} = \frac{EI - W_r}{VI} \quad \ldots\ldots (63).$$

and $$\eta_2 = \frac{W_r}{VI} \qquad (64).$$

we have $$\frac{P_d}{P_m} = \frac{EI + W_r}{EI - W_r} = 1 + \frac{2W_r}{EI - W_r},$$

$$= 1 + \frac{2W_r}{VI} \cdot \frac{VI}{EI - W_r},$$

or $$\frac{P_d}{P_m} = 1 + 2\frac{\eta_2}{\eta_1} \qquad (65).$$

By means of this equation, the equation

$$E = \Psi v = I(R + 2r),$$

and the motor curves, we can find the relation between v and r, for a given value of V.

§ 153. **Example.** A car weighing 15 tons is equipped with two motors whose efficiency is about 80%, the gear and iron losses being about 10%. It is running down a 9% grade with the motors in parallel on a resistance of 2·5 ohms. Each motor's resistance is 1ω; the friction of the track is equivalent to a 1% grade. The test of each motor gave the results below. Find the steady speed down the hill and the current in each motor.

Current (amps.).	Free Speed (M.P.H.).	Pull as motor.	P.D.
40	11·6	756 lbs.	500
50	10·3	1000 ,,	,,
60	9·4	1270 ,,	,,

(Mech. Sc. Trip. 1910).

It is best, in these cases, to draw up a table, thus:—

Data.

I	Ψ	v M.P.H.	P_m pounds.	$E = V - IR$	EI	VI	$W_r = \cdot 1.\ VI$	$EI - W_r$	η_1	η_2	$\frac{P_d}{P_m}$	P_d
40	39·6	11·6	756	460	18,400	20,000	2000	16,400	·82	·1	1·245	942
50	43·7	10·3	1000	450	22,500	25,000	2500	20,000	·80	·1	1·250	1250
60	46·9	9·4	1270	440	26,400	30, 000	3000	23,400	·78	·1	1·255	1595

$V = 500$ volts.
$R = 1$ ohm.

Now, the braking pull required

$$=15 \times 2240 \times \frac{8}{100},$$
$$=2688 \text{ lbs.}$$
$$=1344 \text{ lbs. per motor.}$$

Draw curves of Ψ and P_d against I (Fig. 100).

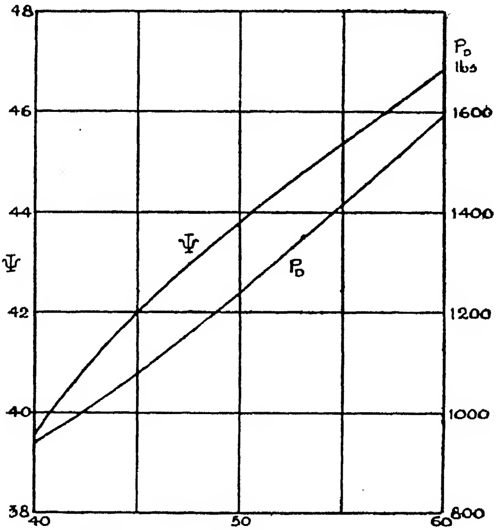


FIG. 100.

We see that, when $P_d=1344$, $I=53$, and when $I=53$, $\Psi=44{\cdot}8$. Hence

$$E=\Psi v=I(R+2r).$$
$$v=\frac{53 \times 6}{44{\cdot}8}=7{\cdot}1 \text{ miles per hour.}$$

SHUNT AND COMPOUND MOTORS FOR ELECTRIC TRACTION.

§ 154. So far we have supposed electric traction to be effected by nothing but series motors, and, indeed, until recently this was so, for, as has been pointed out, for car propulsion they possess all the desirable qualities. Their chief defect is this: When employing the motors as brakes by

reversing and short-circuiting them, all the power generated by them is dissipated by the resistances as heat. If we make them return the power to the line the necessary connections are complicated, and if the E.M.F. fall below the P.D. the polarity will be reversed and motor action start again. With a shunt motor this would not happen since the direction of field flux does not depend on the direction of the current in the armature. Therefore, if we could employ shunt motors as advantageously as series motors, we could effect a saving of power by using the motors as generators when the car is slowing up or running down an incline.

But shunt motors have several disadvantages of their own, from the point of view of traction. In the first place they are unable to take large overload torques for short times, such as are required at starting, and the method of varying the speed by field control is unsatisfactory on account of the small range obtainable. Again, as soon as there is any difference in the sizes of the wheels driven by the two motors, owing to wear, the armatures will have different speeds causing a much more unequal distribution of load, when they are in parallel, in the case of shunt motors than series motors; and the difficulty cannot be overcome by the use of coupling rods, as the two motors on a tram are generally mounted on separate bogies.

We shall now see how these difficulties are overcome in practice, and examine the means of employing regenerative control, as this system of using the motors to supply current to the line is called.*

REGENERATIVE CONTROL.

§ 155. **The Johnson Lundell System.** Here the series-parallel control is employed as far as possible, by providing each armature with two separate windings, each with its own brushes, so that the two windings of each armature can be

* For a more complete description the reader is referred to works on *Electric Traction*, such as that by Messrs. Wilson and Lydall, and *Electric Motors* by Hobart.

put in series or parallel; and as the two motors can be grouped in either way, three arrangements are possible, which gives three running speeds without wasting power in resistances. The fields are provided with two sets of turns, the one set consisting of four parts which can be put in parallel for accelerating, and in series for regenerating, whilst the other set is of low resistance suitable for taking the full armature current as with ordinary "series" turns.

The various stages of starting and accelerating are shown in Fig. 101. At first, (I), the field windings are all in parallel

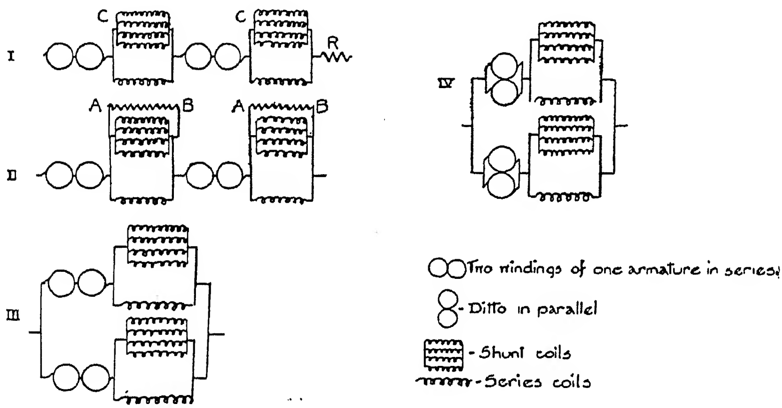


FIG 101

and act as the ordinary field winding of a series motor, the resistance *R* being cut out gradually. The next step is to shunt the fields by coils AB (II), which by diverting the current weaken the field, and the resistance of this coil is reduced gradually so that it takes a greater and greater share of the current which reduces the field still more. Then the two motors are put in parallel (III) with the two windings on each armature in series, and the same process of weakening the fields is repeated, until finally the four armature windings are all in parallel, and again the field is weakened, until the car attains its full speed (IV).

During retardation, when the motors are used as generators for returning power to the line (regeneration) the eight coils

(Fig. 102) are all connected in series to form a shunt across the motors (I. Fig. 102), which now run as compound machines, the series turns being opposed to the shunt turns so as to obtain an equal division of the load and smooth operation. The four

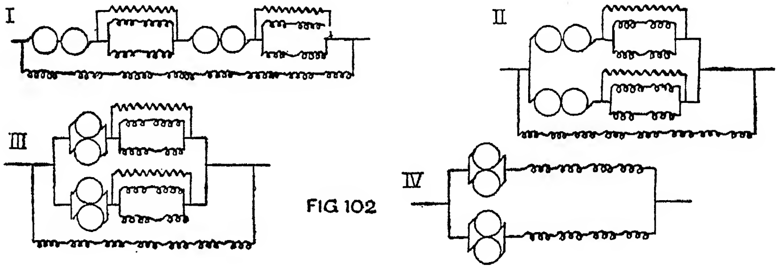


FIG 102

chief stages are shown in Fig. 102, intermediate stages being obtained by an adjustable resistance in the shunt circuit.

§ 156. **The Raworth System.** The motors are compound wound, the shunt field being the stronger, since the purpose of the series turns is only to equalize the load on the two motors. For this system ordinary traction motors can be used, it being only necessary to rewind the field magnets.

The order of operations is shown in Fig. 103.

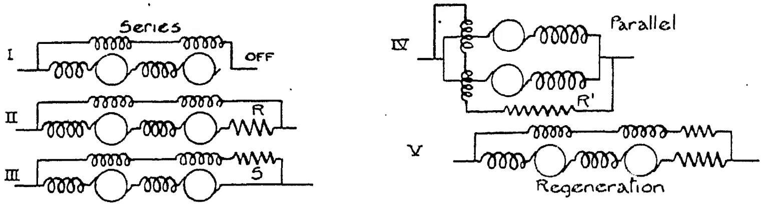


FIG 103

First the shunt coils are connected as in (I), and then the armatures are connected in series with a resistance R (II) which is gradually reduced; this gives one running position. To obtain a greater speed with the series grouping, the fields are reduced by adding a resistance S in the shunt circuit. The connections for running in parallel are shown at (IV), further control being again obtained by means of a resistance R', whilst for regeneration the connections are (V) the same as for acceleration with the motors in series. In the transition stages of (V)

the process is not exactly the reverse of those in (II), as might at first be expected, but the resistance regulation is performed in the same order in both (II) and (V), this being made possible by a special controller.

§ 157. Regenerative systems effect a saving of twenty to twenty-five per cent. in power consumption, but on the other hand the mechanism of control is much more complicated and expensive to install and maintain, so that it must be a matter of time to see whether they are productive of all-round economy, and whether they find general favour.

EXAMPLES.

1. The following are particulars of a 175 H.P. railway motor built by the Société Alsacienne de Constructions Mécanique for the Orleans Railway Co.

No. of poles	4
Armature winding	wave-wound
Total number of conductors	376
Resistance of winding	0·0372
,, ,, field coils	0·0314
,, ,, brush contacts	0·0088
Full-load current	260 A.
Terminal voltage	600 V.
Armature core, cross section	305 cm.^{2}
,, ,, magnetic length	13 cm. per pole.
,, teeth, section	372 cm.^{2}
,, ,, length	4·4 cm.
Air-gap, section at pole face	790 cm.^{2}
,, length	0·5 cm.
Magnet core section	717 cm.^{2}
,, ,, magnetic length	7·5 cm.
Yoke, section	335 cm.^{2}
,, magnetic length	23 cm.
Field turns per pole	28
Leakage factor	1·2.

Supposing that the field magnets are entirely of cast steel that the armature stampings are of iron; that the efficiency of the motor for currents from 150 A. to 450 A. is constant at 92%; use any available iron curves and determine the curves of speed (R.P.M.) and torque (lbs. ft.),

If the gear ratio is 40/24, the diameter of the driving wheels is 40″, and the efficiency of the gears is 94%, deduce curves of tractive effort and speed (M.P.H.).

2. A car has two motors, each of 1·0 ohm resistance it is supplied at 500 V., and is moving on a definite gradient The car is running steadily at 10 M.P.H., taking 100 A. with the

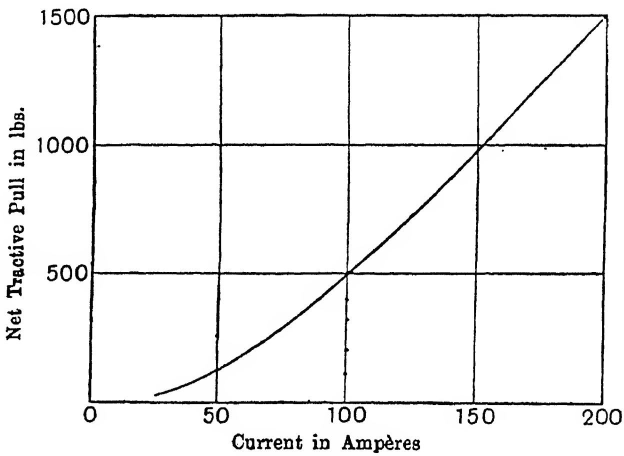


FIG. 104.

motors in parallel. Find the speed with the motors in series and find what connections and series resistances will be required to run at 7 and 2 M.P.H., the speeds being all steady speeds. (Mech. Sc. Trip. 1909.)

3. A tram-car is fitted with two series motors, each of 1 ohm resistance. With the motors in parallel, and a P.D. of 500 V., the car runs on the level at 20 M.P.H., and each motor takes 30 A. Assuming that the rail-pull necessary to propel the car is independent of the speed, find how fast the car will travel when the motors are connected in series with one another and with a resistance of 4 ohms.

When the car is running down-hill, the motors are connected as generators so as to return current to the line. If the rail-pull required to propel the car on the level is 640 lbs., show that when each motor is returning 30 A. to the line, the drag on the rails is about 780 lbs., and the speed 22·6 M.P.H. The motors are in parallel. (Mech. Sc. Trip. 1914.)

4. An electric locomotive is driven by two series motors. The relation between the current taken and the tractive pull exerted by a single motor is given below in Fig. 104. Plot a curve showing the connection between the total tractive pull and the speed of the locomotive (M.P.H.) when the motors are (i) in parallel, (ii) in series. The pressure of supply is 500 V., the resistance of each motor is 2 ohms, and only $\frac{2}{3}$ of the electro-magnetic torque is transmitted to the driving wheels.

If road and wind resistance amount to $60 + \cdot 85\, v^2$ lbs., where v is the speed in M.P.H., find the steady speed attained (i) with the motors in series, (ii) with the motors in parallel. (Mech. Sc. Trip. 1911.)

CHAPTER VI.

MULTIPLE WIRE SYSTEMS AND BOOSTERS.

THE THREE-WIRE SYSTEM.

§ 156. The three-wire system, is used on lighting circuits to save copper, and to supply power to motors required to run at very different speeds. In the case of lighting circuits the power can be distributed at twice the pressure of the lamps, and this reduces the cross section of each conductor to one-quarter its value in the case of two-wire distribution for the same copper loss in the conductors.

§ 157. Suppose we wish to light 100 lamps in parallel at the end of a pair of mains, each taking $\frac{1}{2}$ an ampère, and having a resistance of 200 ohms (hot) with an allowable loss in the mains equal to 10% of the total power. The pressure across the lamps must $=\frac{1}{2}\times 200=100$ volts.

The total current $=100\times\frac{1}{2}=50$ A.

The total power $=100\times 50=5000$ watts.

$\therefore$ allowable loss $=500$ watts.

Let l=length of mains, ρ=specific resistance of copper, s=their cross section. Then $2\rho\frac{l}{s}.(50)^2=500.$

$$\therefore \qquad s=10\rho l.$$

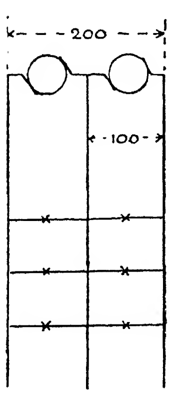


FIG. 105.

Now on the three-wire system, where two lamps are in series (Fig. 105), the total current $=25$ A.

Pressure $=200$ volts.

$\therefore$ Power $=5000$ watts.

Loss $=500$ watts, and

$$2\rho.\frac{s}{l}(25)^2=500$$

$$s=\frac{5\rho l}{2}$$

To make each lamp independent a third wire is run down the middle, usually of half the section of the outside wires, but, even so, there is a great saving in copper.

In the case of motors, the armature may be connected across either the whole pressure, or half the pressure, giving either of two speeds without wasting power in resistances.

§ 158. **Division of Voltage in Three-wire Systems.** The two outside wires are connected to the terminals of the generator, and we must divide the voltage in two, in order to have a point for fixing the middle wire. The most common method is to use a *Balancer Set* (Fig. 106). This consists of two comparatively small similar shunt machines, G and M, with their armatures connected in series, their fields in series and direct coupled Since they are run at the same speed, and excited by the same current, the voltage must be divided equally at A. If the load becomes unbalanced, one machine runs as a motor, the other as a dynamo, the balancer transferring energy from the side which has the less load to that which has the greater as indicated below. Of course a certain amount of current must always flow through the set, even in balance, to overcome the losses in the two machines.

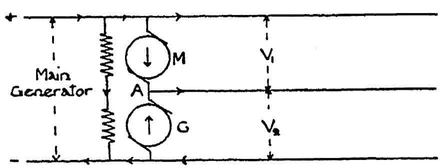


FIG. 106.

Instead of using a motor-generator set, as described above, a single machine having one armature with two windings may be used. Such machines are sometimes called "*dynamotors*"; the two windings have a common field magnet, and each winding of the armature is provided with its own commutator. When the load is different on the two sides, the more lightly loaded side will have the higher pressure difference with the neutral, and the winding which is connected to this side will have motor action, and drive the armature, thus causing an E.M.F. to be generated in the other winding. This E.M.F. will raise the pressure in the side which carries the heavier load.

§ 159. **Equations for Three-wire Systems.** We shall consider first the use of a dynamotor machine (Fig. 107), in which a and b are the two commutators, and f the field winding, of the balancing machine.

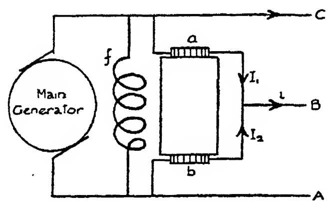


FIG 107

When the load is balanced, i.e. the currents in the two outer mains are equal and no current is flowing in the neutral main.

Let R_a = resistance of each armature winding..

E_o = E.M.F. induced in ,, ..

P_o = rotational losses of the balancer.

V = P.D. between the outer wires.

I_o = current in each armature winding.

Then $V=2E_o+2I_oR_a$ ....(i),

since both sides will have a motor action. Also

$P_o=2E_oI_o.$ ....(ii).

When the load is unbalanced,

let I_1 and I_2 = the two armature currents,

i = the current in the neutral,

then $V=2E_o+I_1R_a-I_2R_a,$ ....(iii).

for, neglecting the effects of armature reaction, the two E.M.F.'s will be the same.

Also $I_1+I_2=i.$ .... (iv)

Also the total output of the motor part must equal the sum of the total rotational losses and the output of the dynamo part, i.e.

$$E_oI_1 = E_oI_2+P_o$$

$$= E_oI_2+2E_oI_o \quad \text{........ by (ii).}$$

$$\therefore \quad I_1=I_2+2I_o,$$

or $I_1-I_2=2I_o.$ ....(v).

The equations (iv) and (v) give

$$\left.\begin{aligned} I_1 &= \frac{i}{2} + I_0 \\ \text{and} \qquad I_2 &= \frac{i}{2} - I_0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(vi)}$$

Also, from (iii) and (v)

$$V = 2E_0 + R_a(I_1 - I_2) = 2E_0 + 2I_0R_a,$$

which is the same as (i). Hence the induced E.M.F., E_0, is not altered by the load, and (vi) shows that the unbalanced current along the neutral main may be regarded as dividing into two equal portions at the balancer.

The P.D.'s across the two pairs of wires are

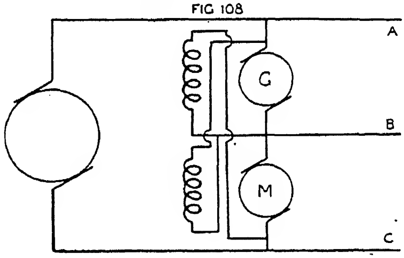
$$E_0 - I_1R_a \text{ between A and B,}$$

and

$$E_0 + I_2R_a \quad \text{,,} \quad \text{B and C.}$$

When a motor-generator set, consisting of two direct-coupled similar machines, is used, as in Fig. 106, with the two-field windings in series across the outer mains, the operation is exactly as described above for a single machine. But better regulation of the voltage is obtained by connecting the field of one machine across the terminals of the other, as in Fig. 108, for the following reasons.

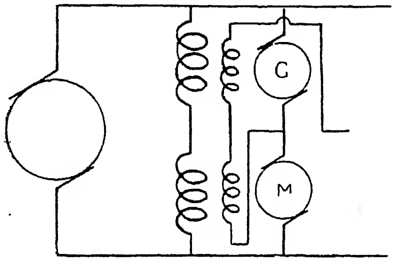
FIG 108



Suppose, Fig. 108, that the load across AB is greater than the load across BC: then the machine marked M will work as motor, and the other, G, as a generator. The pressure across BC will be greater than that across AB, hence the excitation of G will be greater, that of M less, than the corresponding equal excitations when the same total load is balanced, so that the speed of the set will increase. Hence, on account of the increase of speed and excitation, G will have a greater E.M.F. However, this arrangement cannot produce perfect equality of voltage since it depends on an inequality in order that it may work.

A better system is to have the field magnets of the balancer set compound wound, as in Fig. 109, or, better, Fig. 110.

FIG. 109.



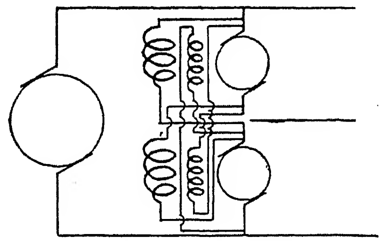


FIG 110

EXAMPLES.

§ 160. **1.** In a three-wire system the outer wires have a resistance of 0·05 ohm each, and that of the neutral is 0·1 ohm. The P.D.'s at the station end are each 100 V., and the loads at the other end are 50 A. and 30 A. Find the P.D.'s at that end. (Special Exam., Cambridge, 1914.)

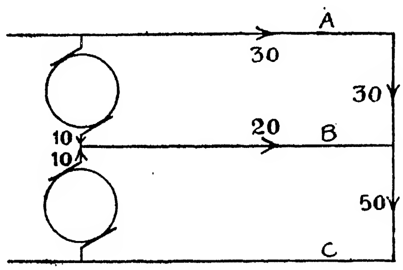


FIG 111

The currents are as shown in Fig. 111.

$$\text{The drop in A} = 30 \times 0{\cdot}05 = 1{\cdot}5 \text{ V.}$$
$$\text{,, ,, ,, B} = 20 \times 0{\cdot}1 = 2{\cdot}0 \text{ V.}$$
$$\text{,, ,, ,, C} = 50 \times 0{\cdot}05 = 2{\cdot}5 \text{ V.}$$

$\therefore$ P.D. between A and B $= 100 - 1{\cdot}5 - 2 = 96{\cdot}5$ V.

and ,, ,, B and C $= 100 - 2 - 2{\cdot}5 = 95{\cdot}5$ V.

2. The pressure across the terminals of the main generator of a three-wire system is 440 V. At 220 V. each machine of

the balancer takes 5 A. armature current to run light, and 2 A. shunt current, and its armature resistance is 0·15 ohm. Between the positive and neutral mains is a load of 50 A.; between the neutral and negative is one of 100 A. Find the pressures across each pair of mains. (Mech. Sc. Trip. 1910.)

We have, in the notation of § 159,

$$E_0 = 220 - \cdot 15 \times 5 = 219\cdot 25 \text{ V.},$$

$$i = \text{the unbalanced current} = 50 \text{ A.}$$

$$\therefore \quad I_0 = 5 \text{ A.}$$

hence

$$I_1 = \frac{i}{2} + I_0 = 25 + 5 = 30,$$

$$I_2 = \frac{i}{2} - I_0 = 25 - 5 = 20,$$

and then

$$V_{ab} = E_0 - I_1 R_a = 219\cdot 25 - 30 \times \cdot 15$$
$$= 214\cdot 75 \text{ V.}$$

$$V_{bc} = E_0 + I_2 R_a = 219\cdot 25 + 20 \times \cdot 15,$$
$$= 222\cdot 25 \text{ V.},$$

and the P.D. across the whole $= 437$ V.

§ 161. **Multiple Wire Systems**.—Speed control of motors. In a motor with given excitation the speed varies directly as the back E.M.F., E other things being equal, and E is nearly proportional to the terminal P.D., since the internal drop is small, hence the speed will be nearly proportional to the applied pressure applied to the armature. We thus see that the three-wire system provides at once an easy method of obtaining two speeds for a motor, by connecting the armature (1) between the neutral and one of the outer conductors, or (2) between the two outers and the field shunt coils being connected between the same pair of mains in both cases, so that the speed in the former case will be about one-half that in the latter.

To gain a greater speed range than this, four-wire, or multiple-wire systems have been devised, two of which will now be described.

(1) **The Crocker-Wheeler System** (Fig. 112), which employs voltages of 40, 120 and 80 as shown, With this system six different voltages, 40, 80, 120, 160, 200, and 240 can be obtained, and six corresponding speeds ; these voltages are in arithmetical progression with a uniform rise without jumps. Actually the speed range obtained is about 1 : 8.

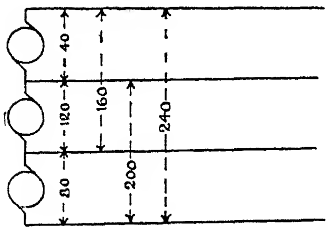


FIG. 112

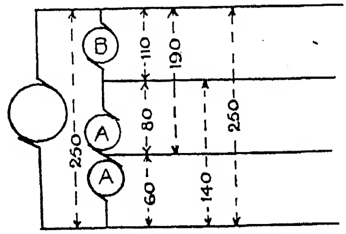


FIG. 113

(2) **The Bullock System** (Fig. 113) also employs four lines at different potentials, giving rise to six available pressures, but in this case they are approximately in geometrical progression, namely, 60, 80, 110, 140, 190, 250, which give a speed ratio of about 1 : 5.

§ 162. With these multiple voltage systems it is possible to use three separate generators giving the three different voltages, or to use balancer sets, but the latter is the usual arrangement since it is cheaper. The systems, although economical on account of the motors not wasting so much power in resistances, and being more productive when used for driving machine tools on account of the flexibility of speed, are expensive on the score of auxiliary machinery and mains, and on account of this extra expense, and the extra complications involved, it is only desirable to employ such systems under exceptional circumstances.

§ 163. The change over from one voltage to another is effected by a controller, consisting usually of a cylinder having projecting contacts on its surface. By rotating the cylinder, connexion is established between the armature of the motor and the different lines in turn. In Fig. 114 the surface of this

cylinder is supposed to be developed into a plane, with the contact pieces shown black, while A, B, C, D, are fixed contact points connected to the mains and EF contacts for the armature

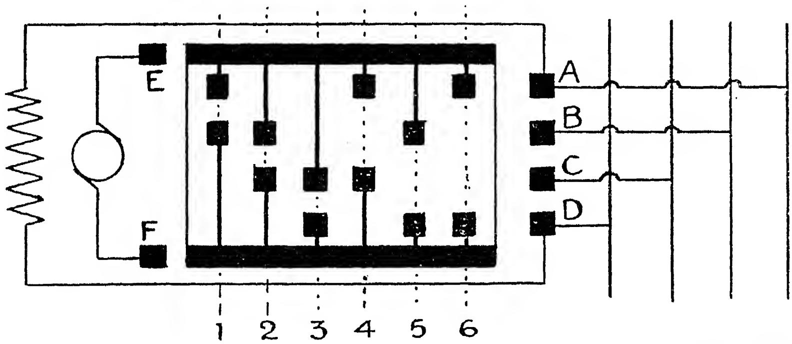


FIG. 114.

circuit. The field is connected permanently across the highest voltage. The running positions of the motors are obtained when the lines (1), (2), (3), etc., are on the line EF.

§ 164. **Equations for Four-Wire Systems.** (Fig. 115.)

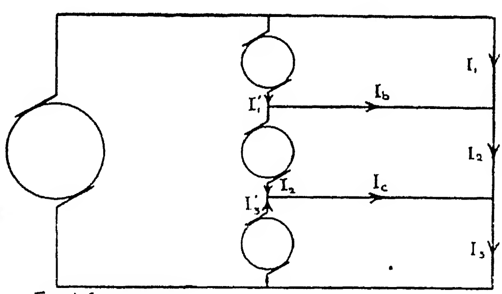


FIG. 115.

Let E_1, E_2, E_3 be the induced E.M.F.'s in the different armatures of the balancer, and suppose the two top machines in the figure are acting as motors, the other as a generator.

Let P_0 = the rotational losses of the balancer set.

I_0 = the no-load current taken by it.

Then, when the load is balanced, and all the machines are acting as motors, we have

$$P_o = (E_1 + E_2 + E_3) I_o. \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}.$$

When the load is unbalanced, with the currents as shown, we have:

$$I_2' = I_1' - I_b \qquad I_2 = I_1 + I_b$$
$$I_2' + I_3' = I_c \qquad I_3 = I_2 + I_c,$$

whence

$$\left.\begin{array}{l} I_2' - I_1' = I_1 - I_2 \\ I_2' + I_3' = I_3 - I_2 \end{array}\right\} \quad \ldots\ldots\ldots\ldots \text{(ii)}.$$

Also, since the output of the two motors = that of the generator + the rotational losses,

$$\begin{aligned} E_1 I_1' + E_2 I_2' &= E_3 I_3' + P_0 \\ &= E_3 I_3' + (E_1 + E_2 + E_3) I_0 \quad \ldots\ldots \text{(iii)}, \end{aligned}$$

by (i).

From (ii) and (iii), writing $E_1 = \alpha E$, $E_2 = \beta E$, and $E_3 = \gamma E$, we obtain

$$\left.\begin{array}{l} I_1' = I_0 - (\beta + \gamma) I_1 + \beta I_2 + \gamma I_3 \\ I_2' = I_0 + \alpha I_1 - (\gamma + \alpha) I_2 + \gamma I \\ I_3' = I_0 + \alpha I_1 + \beta I_2 - (\alpha + \beta)_3 I_3 \end{array}\right\},$$

giving the currents taken by each armature of the set.

BOOSTERS.

§ 165. The object of a booster is to keep the load on the main generators in a generating station as nearly constant as possible, in spite of large fluctuations in the current demanded by the load. In lighting circuits there are certain hours of the day when there is a much larger demand for current than the average of the current taken over a whole day, and this demand can be met by running extra generators during the required time, for the peaks in the current-time curve occur at more or less known hours. But in traction stations this is not so, and the trouble is enhanced in small and medium-sized stations, especially in hilly districts. The peaks may occur at any time, and it is necessary to keep a large enough plant running to cope with them whenever they come on. At the

same time there are certain hours, such as in the night or early morning, when only a few cars have to be run. This means that the main generators will have to be worked at a point far below their best efficiency, which results in a great loss, or else that a special small unit must be installed with a consequent large capital outlay in plant, building, etc.

Obviously, then, what is wanted is some form of apparatus which will automatically help the main generators when required, and this condition is fulfilled by the automatic reversible booster, which we will now describe.

AUTOMATIC REVERSIBLE BOOSTER.

§ 166. The booster set consists of a small generator or "booster" direct coupled to a shunt wound motor, and a battery. The motor is run off the bus bars, and the armature of the booster is connected in series with the battery between the bus bars, as shown in Fig. 116. Circuit breakers, K, are

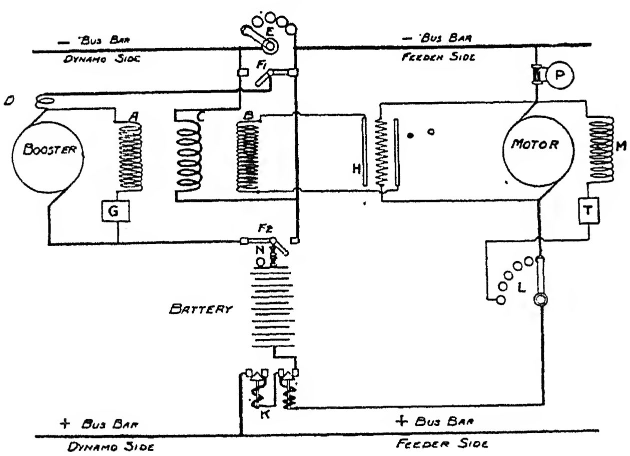


Fig. 116 (by courtesy of the Lancashire Dynamo & Motor Co.)

provided in both the booster-battery and motor circuits, so arranged that the motor cannot be cut out when current is passing through the booster, which would cause the latter to run up to a dangerously high speed. When the left or "battery" breaker comes out it leaves the motor running on the battery, although cut off from the bus bars; while, if the motor breaker (right hand) comes out, the motor is left running on the bus bars.

The booster has four field coils, A, B, C, D, as shown in the diagram. The coil A is connected across the booster brushes, so that it is excited by the difference of voltage between the battery and bus bars. Suppose the battery be 10 volts higher than the bus bars, then the A coil will cause the booster to give 10 volts against the battery; on the other hand, if the battery be 10 volts lower than the bus bars, A will cause the booster to act in favour of the battery, and add 10 volts. Hence, in every case, the battery volts and booster volts together equal the bus bar volts.

The coil D consists of a few turns in series with the armature to compensate for armature reaction and armature resistance.

The coil B is constant in its action, being excited under the bus bar voltage, and controlled by a potentiometer resistance H. When the booster is working in the ordinary way, with the generators running, the excitation of this coil is in such a direction as to cause the current to charge the battery.

The coil C is arranged in series with the generators, so as to take the main current or a portion of it, adjustment being made by the series diverter E. The current passes through C and so regulates the booster's E.M.F. as to allow current to flow from the battery into the mains or conversely. Hence B and C are opposed to one another.

Now, suppose the booster is running with the generator, and that B and C are adjusted, say, for 100 ampères dynamo current: at this load they will neutralize one another, and the booster will neither charge or discharge the battery. Suppose an excess load comes on the plant: the generator will supply a little more current, which will go through C, enabling it to overcome B's action, and so draw the excess current from the battery. Similarly, if the load fall below 100 ampères, a little less current will flow through C, and the superior strength of B will cause current to charge the battery.

§ 167. **Relay Boosters** have been introduced in order to reduce the cost of a booster with its three-field windings. With this arrangement the booster has only one field winding A (Fig. 117), and a carbon pile regulator connected as shown,

CC being columns of carbon blocks. The resistance of these columns depends on the degree of pressure between the blocks, and this is automatically varied by the solenoid B, the plunger of which turns the lever LM about the pivot D, thus reducing the pressure on one column and increasing that on the other. With the normal load these pressures are equal, and no current flows through the field A, but, when the load is increased or

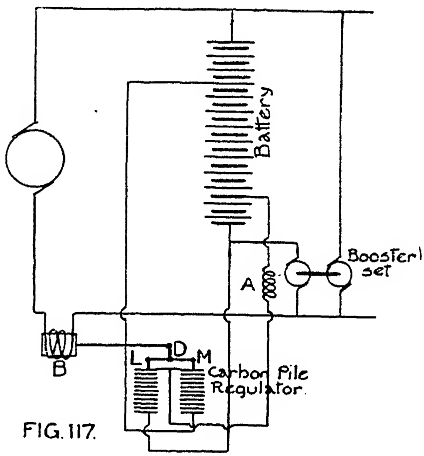
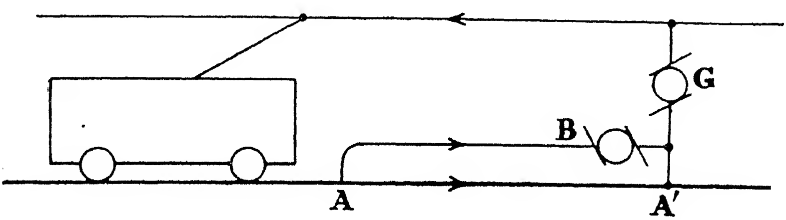


FIG. 117.

decreased the current in the coil B is changed, and so the resistances of the carbon piles become unbalanced, and current flows, one way or the other, through the booster field as required.

§ 168. **Example.** The figure represents diagrammatically an electric tramway. The cars, all of which are beyond A,



take a total current of 250 A. from the generator G. The current returns by the rails as far as the point A, distant 3 miles from the generator, where some of it is diverted into a negative feeder AB. An independently driven booster B, situated in the power station, is included in the feeder. The resistance of

the rails is 0·02 ohm per mile, that of the feeder is 0·08 ohm per mile. Find what E.M.F. must be developed in the booster in order that the difference of potential between A and A^1 may be 6 volts. (Mech. Sc. Trip. 1914.)

Let I_1 = the current in the booster.
I_2 = „ „ „ rails, AA^1.
E = the E.M.F. of the booster.

The resistance of the feeder $= 3 \times \cdot 08 = 0 \cdot 24$.
„ „ „ rail $= 3 \times \cdot 02 = 0 \cdot 06$.

Then we must have

$$6 = I_2 \times 0 \cdot 06 = I_1 \times 0 \cdot 24 - E,$$

and

$$I_1 + I_2 = 250.$$

Solving these equations gives

$$\begin{cases} I_1 = 150 \text{ A.} \\ I_2 = 100 \text{ A.} \\ E = 42 \text{ V.} \end{cases}$$

EXAMPLES.

1. A three-wire distributor, 200 yards in length, has the feeding point at one end, where the pressures between the neutral and each outer are maintained at 200 V. Currents are tapped off as below:—

Distance from feeding point, yds.	20	40	70	100	120	160	200
Tappings on + side, amps.	—	10	15	—	12	25	—
„ „ − „ „	20	—	—	10	—	—	25

The resistance of the outer wires is 0·4 ohms per 1000 yds., and that of the neutral 1·6 ohms per 1000 yds. Find the voltage between the neutral and each outer main at the distant end of the distributor. (Special Exam., Cambridge, 1908.)

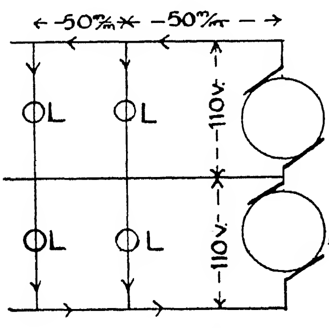


FIG. 119.

2. Twenty arc lamps, L, L, . . ., each of which takes 10 A., are arranged at intervals of 50 metres, as in Fig. 119. The section of each outer conductor is 1 cm.^{2} and

that of the middle conductor $\frac{1}{2}$ cm.^{2} Find the P.D. between the terminals of each of the two most distant lamps:

(i) When all the lamps are burning.

(ii) When the five lamps nearest the generator, on one side, are turned out.

The resistance of a copper wire 1 cm.^{2} section is 0·16 ohms per kilometre. (Mech. Sc. Trip. 1908.)

3. The balancer for a certain distribution on the Bullock system has armature resistances of 0·12, 0·16, 0·22 ohms, for the 60 V., 80 V., and 110 V. elements respectively. The no-load current is 8 A. If there be loads of 40 A., 30 A. and 100 A. respectively, what are the currents in the three armatures, and the P.D.'s between the three wires?

4. The trolley wire of a tramway, which is distant from the generating station, is supplied by a single feeder F, Fig. 120.

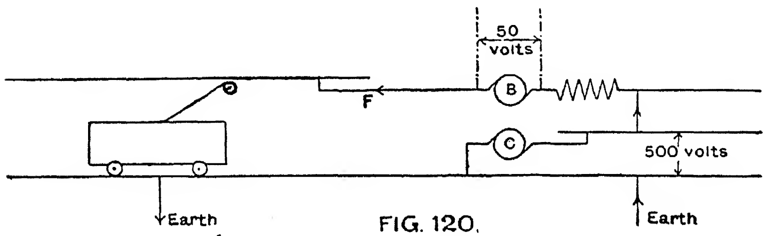


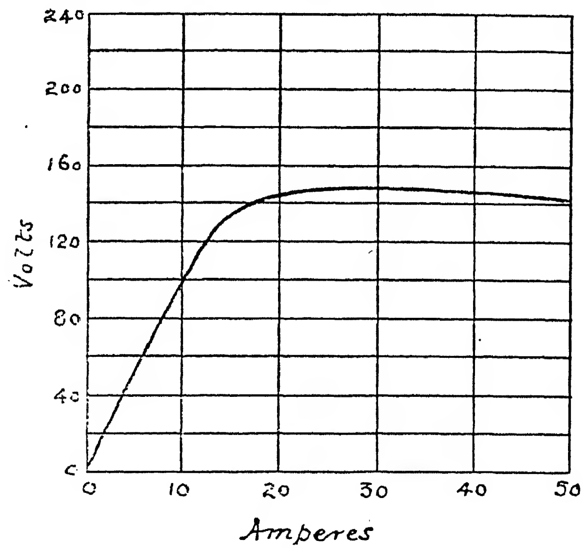
FIG. 120.

A series wound booster, B, is placed between the feeder and the main bus-bar. The booster is driven at an approximately constant speed by the shunt wound motor C (which is connected to the main 'bus-bars), and is so proportioned that it adds to the 'bus-bar potential an amount roughly equal to the drop in the feeder (which is about 50 V. at normal load) and keeps the potential on the line nearly constant. While the cars are running, taking full current, and the booster is giving the full potential of 50 volts, the circuit of the motor C is accidentally opened: What happens to the booster if the cars continue to take their full normal current? (Mech. Sc. Trip. 1907.)

MISCELLANEOUS EXAMPLES.

1. A 2-pole shunt dynamo has 6000 turns of wire, 2 mm. diameter, on the field magnets, and gives at 500 R.P.M. an E.M.F. of 200 V. It is desired to run it at 250 R.P.M., the winding of the field magnets being altered to give the same flux density and the same loss of energy in the winding as before. What will be the number of turns and diameter of wire required? (Special Exam., Cambridge, 1912.)

2. The sketch below shows the external characteristic of a series dynamo when running at 700 revs. per minute. The armature and field resistances, taken together, being 1·82 ohms, deduce from the curve the total characteristic, that is the curve connecting the E.M.F. produced by the armature with the current. (Special Exam., Cambridge, 1909.)



3. A dynamo armature is running with a current of 10 A., and a P.D. of 105 V. If the resistance be 0·4 ohm, the speed 1200 R.P.M., and the number of armature wires 500, find the flux passing through the armature. (Special Exam., Cambridge, 1905.)

4. A dynamo is run separately excited, and without load, at 500 R.P.M., and the following values of the E.M.F. and exciting current are obtained:

Field Current	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
E.M.F.	100	190	260	310	355	395	425	455	480	500.

What must be the field current if the E.M.F. is to be 300 V. at 350 R.P.M.? (Special Exam., Cambridge, 1912.)

5. A series dynamo has an internal resistance of 1 ohm and on short circuit requires a field current of 6 A. to give an armature current of 20 A. The saturation curve is given approximately by :—

I_f	0	2·5	15	20
E	0	20	94	104.
	straight.		straight.	

Find the E.M.F. and P.D. when running as a series dynamo delivering 20 A.

6. The armature of a 2-pole shunt dynamo has 300 conductors, and the full load current is 30 A. If the brushes have a lead of 20°, find the extra field current required at full load to compensate for armature reaction, if there be 500 shunt turns, and if the demagnetizing turns only are considered. (Trin. Coll., Cambridge, 1913.)

7. A shunt dynamo has armature and shunt resistances of 0·12 and 70 ohms respectively. The saturation curve and the short-circuit characteristic are as follows:

Saturation curve.		*Short circuit characteristic.*	
E	I_f	I_a	I_f
0	0	0	0
10	0·04	40	0·22
20	0·08	80	0·44
204	2		
213	2·5		
220	3		
227	3·5		

Predict the external characteristic up to a full-load armature current of 80 ampères.

8. A shunt dynamo running at 1200 R.P.M. is giving a pressure of 250 V., and the armature current is 10 A., its resistance being 1 ohm. The area of the polar face is 2000 cm.^{2}, and the air-gap induction is known to be a little over 5000. If the number of commutator sections is 32, find the number of conductors per section, and the true value of the flux. (Inter-Coll. Exam., Cambridge, 1906.)

9. An armature with 100 peripheral wires is rotating at 1200 R.P.M., giving an E.M.F. of 100 V. The induction is 4000. Find the area of the polar faces.

10. The field coil of a dynamo has a resistance of 100 ohms at 15°C. The resistance increases by 0·4 ohm per 1°C. rise, and the coil loses heat by radiation at the rate of 7 watts for every degree by which its temperature exceeds 15°C. Find what temperature the coil reaches when an E.M.F. of 250 V. is continuously maintained across it. (Mech. Sc. Trip. 1914.)

11. A dynamo has 10 poles, each wound with 300 turns, all the coils being in series. The P.D. on the fields is 150 V. and the total flux in each pole is 10^6 lines. The effect of opening a switch in the exciting current circuit is to reduce the current to zero at a uniform rate in $\frac{1}{10}$ second. Trace the variations of E.M.F. across the terminals of the switch, and show that it reaches a maximum of 450 V. It may be assumed that the machine has a straight characteristic. (Mech. Sc. Trip. 1914.)

12. The following particulars refer to a 6-pole, 250 *k*w. D.C. generator. The P.D. is 250 V. at 500 R.P.M.

Armature.	Diam. outside	90 cm.
	Gross core length	25 ,,
	Internal diameter	51 ,,
	6 ventilating ducts, width..	1 ,,
	84 slots, each 4 × 1·2 cm.	

Armature Winding. Single, lap, 6 conductors per slot.
Section of bare conductor 3·2 × 12·5 mm.

Air-gap. 5 mm.
Pole arc /Pole pitch = 0·6

Magnet Core. Diam. 27 cm.
Axial length 26 cm.

Yoke. External diam. 171 cm., internal 145 cms.
Width 58 cms. Rectangular section.

Calculate (1) the current density in the armature conductors, (2) the resistance of armature between brushes, (3) the armature flux per pole at full-load, (4) the flux density at the roots of the teeth, (5) the armature copper losses, (6) the no-load saturation curve.

For the magnetic properties of iron use any typical curves.

13. A 2-pole series motor has 100 conductors on its armature and a resistance of 1 ohm. It is taking current at 200 V. Fill in the following table:

Current	20	30	40
Air-gap flux	$17 \cdot 10^6$	$20 \cdot 10^6$	$23 \cdot 10^6$
Speed			

(Special Exam., Cambridge, 1911.)

14. A shunt motor is supplied with a current of 25 A. at 200 V. Armature resistance $=0 \cdot 2$; shunt resistance 55 ohms; speed 700 R.P.M. Find the back E.M.F. and the gross torque. If the total armature losses are 200 watts, find the nett shaft torque. (Special Exam., Cambridge, 1910.)

15. A 2-pole motor, with 80 peripheral wires on its armature and a pole flux of $9 \cdot 8 \times 10^6$ lines, has an armature current of 200 A. Find the gross torque.

16. A dynamo has an armature 10″ diam., and 12″ long. The air-gap induction is 6500, and each polar arc is 120°; armature wires $=136$; speed $=845$ R.P.M. Find the E.M.F. What torque will be required to drive it when delivering 100 A., if that required to overcome friction and iron losses is 2 lbs. ft.? (Special Exam., Cambridge, 1913.)

17. A dynamo when run at 600 R.P.M. with a current of 1 A. in the shunt, which has 2000 turns, gives an E.M.F. of 100 V. The armature turns are 200. The shunt resistance is 100 ohms.

Find the reluctance of the magnetic circuit, and the minimum exciting speed. (Inter-Coll. Exam., Cambridge, 1903.)

18. A 4-pole wave wound series motor has a resistance of 1·2 ohms. It runs at 660 R.P.M., taking 50 A. at 500 V. The nett shaft torque = 300 lbs. ft.; the lost torque due to friction = 2% of gross torque, and the remaining lost couple is equally divided between hysteresis and eddies. A resistance of 2·4 ohms is put in series, and the load adjusted so that 50 A. again flow. Find the new couple. (Mech. Sc. Trip. 1909.)

19. By a statical experiment it is found that a current of 20 A. in a certain series motor produces a torque of 400 lbs.ft. If the P.D. = 500 V., and the series resistance is 3 ohms, find the speed of the motor at which this torque will be developed. The effects of hysteresis and eddy currents are to be neglected. (Mech. Sc. Trip. 1911.)

20. The data below refer to a shunt motor's saturation curve at a given speed, and it is found that, with the armature short-circuited, a current of 0·52 A. is required in the shunt for the full-load armature current of 80 A. The resistance of the armature is 0·15 ohm; the resistance of the shunt is adjusted so that, when the P.D. is 220 V., the motor's ideal speed, with no rotational losses and no load, is the same as that for which the saturation curve was obtained. Assuming the short circuit characteristic and the first part of the saturation curve to be straight, find the number of series field turns, of total resistance 0·1 ohm, required in the armature circuit if the motor is to have the same speed at no load and full load. The number of shunt turns is 2000. Find also the greatest percentage fluctuation of speed. (Mech. Sc. Trip. 1912.)

E.M.F.	0	50	200	209	215	218	220
Field Current	0	0·5	3	3·5	4	4·5	5

21. A shunt motor has a drop of pressure of 10% in the armature at full-load: the magnetic circuit is such that a 25% fall in exciting current produces a 10% change in flux. At full-load the armature de-magnetizing turns are 15% of the field ampère-turns due to the shunt excitation. Find the ratio of full-load speed to no-load speed, the P.D. being the same. (Mech. Sc. Trip. 1919.)

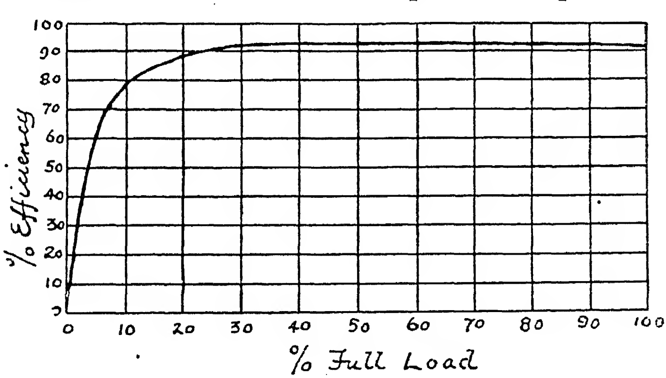
22. A motor has an air-gap such that the saturation curve is practically straight up to 200 volts, under which pressure it runs light at 1200 R.P.M. Its shunt has a resistance of 40 ohms. The field turns number 5000, the armature wires 400. Assuming that on breaking the main circuit the deceleration is at first uniform at one R.P.S.P.S., find the field current at the end of five seconds. (Mech. Sc. Trip., B, 1915.)

23. Find the approximate number of conductors required for a 10-pole lap wound armature to give 550 volts at 240 revs. per min., the total flux through each pole face being $3{\cdot}5 \times 10^6$ lines. (Inter-Coll. Exam., Cambridge, 1919.)

24. A series motor, running at 240 R.P.M., is taking 50 ampères at 440 volts P.D.: its resistance is 1·2 ohms: the lost torque is 8% of the gross. Find the nett torque, and the efficiency. If the lost torque is accounted for by an approximately constant current, find the current and efficiency when the output is 19 H.P. (Mech. Sc. Trip. 1919.)

25. A shunt motor takes a current of 7·5 A. at 100 V.; its output is 0·8 H.P. Find its efficiency. The resistance of the field is 200 ohms, that of the armature 1 ohm. Find the copper loss, and the total of core and rotational losses. (Special Exam., Cambridge, 1905.)

26. The sketch below shows the efficiency curve of a 50 *k*w. shunt motor, shown with respect to the percentage of



full power supplied to the motor For the purpose of testing a dynamo, this motor is used to drive the dynamo, the shafts being coupled direct. On a current of 25 ampères at 200 volts

being supplied to the motor, it is found that the dynamo gives out 36 ampères at a pressure of 99·5 volts. What is the efficiency of the dynamo at this load? (Special Exam., Cambridge, 1909.)

27. A shunt dynamo is delivering 10*k*w. at 100 v. to an external circuit. The armature resistance is 0·04 ohms, and the shunt resistance 50 ohms. The losses due to hysteresis, eddy currents and friction may be taken as 3% of the whole power supplied to the dynamo. Find the efficiency and the torque required at 300 R.P.M. (Special Exam., Cambridge, 1911.)

28. A shunt motor has an armature resistance of 0·1 ohm, and shunt resistance 50 ohms. It works at 200 V. The armature current at no load is 4 A. Find the efficiency and output when the motor takes 100 A. (Special Exam., Cambridge, 1913.)

29. The efficiency of a shunt motor is 85%. It is developing 5 H.P. at 100 V. The shunt resistance is 40 ohms, and the armature resistence 0·1 ohm. Tabulate the various losses. (Special Exam., Cambridge, 1914.)

30. A shunt dynamo runs a series motor through two mains, each of resistance 0·01 ohm. Resistance of motor 0·03 ohm; dynamo armature 0·02 ohm, dynamo shunt 10 ohms. The friction and iron losses amount to 1 *k*w. in each machine. Find the overall efficiency of the whole plant, when the motor is developing 30 B.H.P. The P.D. of the dynamo is 100 V.

31. A rated motor is delivering 1 *k*w. to a pair of direct coupled shunt machines, in which the circulating current is 50 A. at 200 V. Find approximately the efficiency of each machine.

32. A series motor, loaded, takes 50 A. at 150 V., and runs at 600 R.P.M. Its resistance is 0·2 ohm. It then has its field separately excited with 50 A., and by means of an auxiliary motor it is found that the armature of the series motor takes 400 watts to drive it at 600 R.P.M. What are the gross torque, the nett torque, and the efficiency? (Inter-Coll. Exam., Cambridge, 1913.)

33. A compound wound lighting generator gives the following open-circuit characteristic:

Current in each field winding.	Volts at 250 R.P.M.
10	84
12·5	102
15	114
17·5	124
20·0	131

Up to 10 amps. the characteristic is a straight line. There are 400 shunt turns in each field winding. When short-circuited and separately excited with a current of 4 amps. in the shunt winding and run at 250 R.P.M. the machine gives the full-load current of 1000 amps. The power required to turn the armature under these conditions exceeds by 3000 watts that required to drive it at the same speed when unloaded and unexcited. Find how many series turns must be put on, in order that the machine may give 115 terminal volts at 250 R.P.M. at no-load and at full-load, with the same setting of the brushes. With automatic adjustment would you provide in order to secure precisely this amount of compounding with an integral number of series turns? (Mech. Sc. Trip. 1913.)

34. Compare the weights of copper in the mains of a two-wire system, and a three-wire system for the same working voltage if the neutral wire is to have $\frac{1}{3}$ the section of each of the outer wires.

35. A two-pole machine when fully loaded in the armature, which has 300 wires on it, and with a backward lead of 30° gives the same P.D. as at no-load. When arranged to work with no load, what approximate number of series turns will be required per pole? What difference is there between the two conditions of operation? (Mech. Sc. Trip., B., 1915.)

36. Explain the operation of a "booster" for direct-current work. It is required to charge a battery of cells demanding 140 volts: show the arrangement required (1)

when the source of energy is at 110 volts, (2) when it is at 220 volts. In each case sketch in the direction of the various currents for the cases of running light and charging. (Mech. Sc. Trip., B., 1915.)

37. A shunt wound dynamo separately excited and run at its normal speed with the armature on open circuit gives the following readings:

Exciting current	6 amps.	7 amps.	0·25 amps.
E.M.F. in armature	200	208	25

With the armature short-circuited and the brushes in the normal position, an armature current of 300 amps. is obtained when the exciting current is 0·60.

The resistance of the armature being 0·036 ohms and the resistance of the shunt being 30 ohms, find the external P.D. corresponding to 250 ampères in the armature.

If the voltage on open circuit is 200, and this is maintained the same when 300 amps. are flowing in the armature by means of series turns, calculate the value of the shunt resistance and the number of series turns required. (Mech. Sc. Trip. 1915.)

38. A shunt dynamo delivers 100 ampères on short-circuit when excited with $\frac{1}{4}$ ampère. The armature resistance is 0·04 ohm, and to produce 4 volts on open circuit requires an excitation of 0·1 ampère. The field has 8000 turns. The saturation curve is such that to produce a rise of 4 volts from the required P.D. on open circuit demands an extra field current of 0·8 ampère. Find the series turns required to compound correctly with an armature current of 100 ampères. (Mech. Sc. Trip. 1916.)

39. A shunt motor is working with a constant P.D. of 220 volts; the shunt resistance is 40 ohms, the armature resistance is 0·16 ohm, the no-load armature current is 2 ampères. When the output is $13\frac{1}{2}$ H.P., find the current taken and the efficiency. (Inter-Coll. Exam., Cambridge, 1919.)

40. A series motor gives a nett torque $=10(I-10)$lb.ft.: it is taking 65 ampères at 500 volts, with an efficiency of 82%: the resistance is 1 ohm. Deduce the speed, the value of Ψ, and the lost torque. (Inter-Coll. Exam., Cambridge, 1919.)

41. A Hopkinson-Kapp test is performed on two similar machines coupled together.

The energy supplied to the whole plant consists of a current of 40 ampères at a pressure of 220 volts.

The armature resistance of each machine is 0·05.

The motor field current is 6 amps.

The motor armature current is 200 amps.

The dynamo field current is 7 amps.

Find the efficiency of each machine under the test. (Mech. Sc. Trip. 1915.)

42. A shunt motor whose terminal P.D. is 200 volts and whose armature and field resistances are respectively 0·02 and 60 ohms, supplies a series motor of resistance 0·05 through mains of joint resistance 0·1 ohm. If the mechanical efficiency of each machine is 94%, find the current in the mains and the overall efficiency when the motor's output is 30 H.P. (Mech. Sc. Trip. 1916.)

43. A small series dynamo gives a P.D. of 40 volts when delivering 12½ ampères, its total resistance being 0·65 ohm. The saturation curve at the same speed was as given below: the brush lead was 30°. Find the ratio of armature turns to field turns. (Inter-Coll. Exam., Cambridge, 1919.)

Current	7	9	11	13
E.M.F.	40	46	50	52½

44. A car weighing 20 tons is ascending a slope of 1 in 30 at a speed of 6 miles per hour, propelled by two motors in series directly across the mains. The current taken is 50 amps. and the pressure across the mains is 500 volts.

The motors are then put in parallel with a resistance of 1·0 ohm in series with them. Calculate the steady speed which will be acquired by the car up this same slope, and determine the total efficiency of this method of propulsion in the two cases. The resistance of each motor is 1·5 ohm. (Mech. Sc. Trip. 1915.)

45. The machine of question 5 runs at 750 R.P.M. at no-load when supplied at 100 volts, the current taken being so small that drop and reaction can be neglected. Find the speed at which it will run when the load is such that the armature current is 100 ampères, assuming that the saturation curve is straight throughout the range concerned. (Mech. Sc. Trip. 1916.)

46. A shunt motor takes a current of 52 ampères at 200 volts: the shunt resistance is 100 ohms: the armature resistance is 0·2 ohm: the output is 8·9 kilowatts: $\frac{1}{3}$ of the rotational loss is due to eddy currents. The shunt is put with two halves in parallel, and half the P.D. is applied to the whole machine, taking the same current. Compare the output and efficiency with the former values. (Mech. Sc. Trip. 1919.)

47. A 10-ton car equipped with two series motors each of 1 ohm resistance, works on 500 volts P.D. It is running steadily on a 1 in 11 grade (for which the resistance is 25 lbs. per ton) with the motors in series but no extra resistance. It comes to a level part of the track and proceeds at the same initial speed but with the motors put in parallel and a series resistance. Find that resistance and the acceleration. If the constant speed is 5 miles per hour, find the efficiency in the last case.

The pull per motor with 50 ampères = 1000 pounds, and with 60 ampères = 1270 pounds. (Mech. Sc. Trip. 1919.)

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